

Multi-period financial stress testing: an integrated risk view

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AI for Time Series

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- ❑ **Bank capital adequacy**
- ❑ Credit risk models and parameters
- ❑ Multi-period models
- ❑ Macro-economic panel data models
- ❑ Implementation and application

Banks are required to hold capital buffers to fulfill their role in the economy

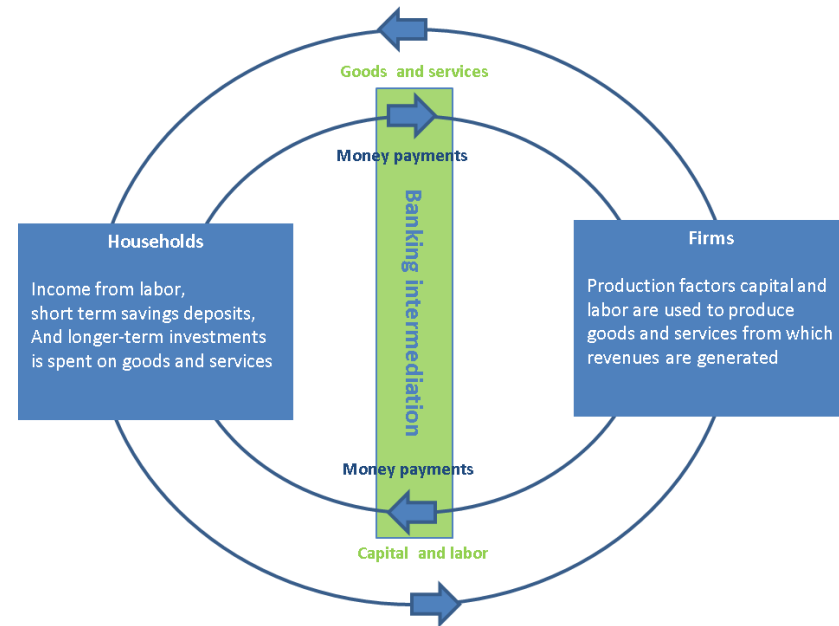
Financial intermediation role of banks in the economy

□ Role of banks in the economy as financial intermediation:

- Liquidity transformation: short term savings account → medium/long term corporate investments
- Risk transformation: risk free savings account → risky corporate investment
- Information transformation: generic product with zero risk → specialized investments

□ Banks receive and provide cash in the economy

- These intermediation cash flows are subject to financial risks
- A capital is needed to have a buffer absorbing these uncertainties
- The capital level is decided by management
- Basel regulation defines minimum capital levels to avoid economy disruptions from bank failures



Bank are required to have adequate levels of capital in both base and stressed forward looking scenarios

Stress testing is an important tool

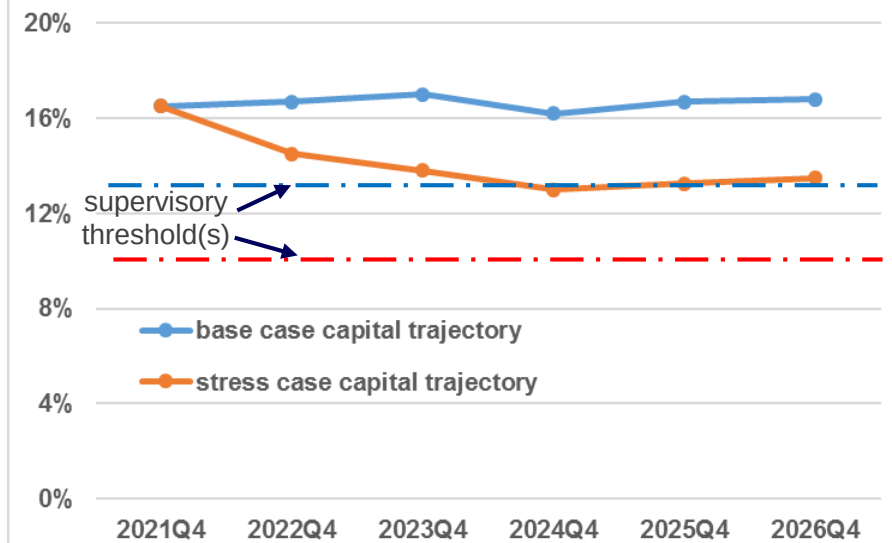
- ❑ Banks are required to have sufficient spot capital in relation to the risks taken. Capital adequacy is monitored by means of a number of Key Risk Indicators.
 - ❑ The Total Capital Ratio (TCR) is a regulatory Key Risk Indicator* that compares the risk levels (Risk Weighted Assets, RWA) for credit, market and operational risks with the regulatory total capital level.
 - ❑ The Total Capital Ratio may not be lower than the supervisory threshold (at least 8%). The Total Capital Ratio should be above the supervisory thresholds
 - spot
 - base case 3-5 year forward
 - stress case 3-5 year forward
- These thresholds depend, a.o., on the macro-economic conditions.

Total Capital Ratio

$$TCR = \frac{\text{Capital}}{\text{Risk Weighted Assets}}$$

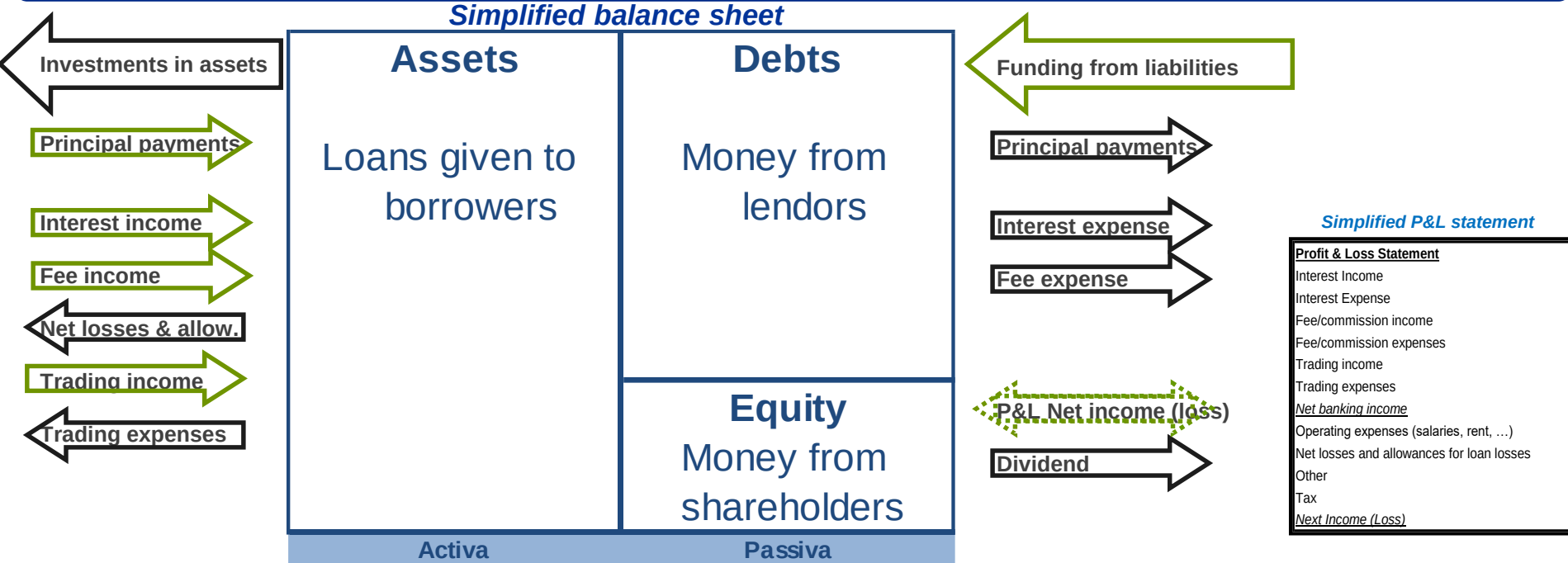
$$RWA = RWA_{\text{credit risk}} + RWA_{\text{market risk}} + \dots \\ \dots + RWA_{\text{operational risk}}$$

Minimum Capital Ratio levels



Models are required to capture the main components in the behaviour of the banks balance sheet and capital ratio

Cash flows : schematic overview



In their intermediation role, the bank is subject to uncertain cash flows resulting from credit, market, funding & liquidity risk. On top of operational expenses, other uncertainties result from operational risk, the risk of wrong/fraudulent operations.

Depending on the accounting rules, the valuation of the financial assets is subject market volatility, though does not generating a real cash flow volatility.

The bank's capital needs to absorb needs to absorb the uncertainties in the cash flows and valuations to avoid the bank defaulting on its liabilities.

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Credit risk and credit quality are a key driver of the capital adequacy

Credit risk is a main risk in banks

Credit risk is the risk that customers default, that is fail to comply with their obligation to service debt. ...

- **Default Risk:** Your debtor does not pay you ...
 - ... what she owes you (capital and/or interests; latent value)
 - ... at the right moment (never, delayed payment)
 - ... because of financial troubles (bankruptcy, rescheduling), she is not in the mood for it (fraud, legal disagreement ...), technical problem
- **Loss Risk:** in case of a default event, you recover only a fraction on the outstanding (capital and/or interests; latent value)
- **Exposure Risk:** the amount to be repaid may change (additional drawings on a credit line or credit card; market factors).

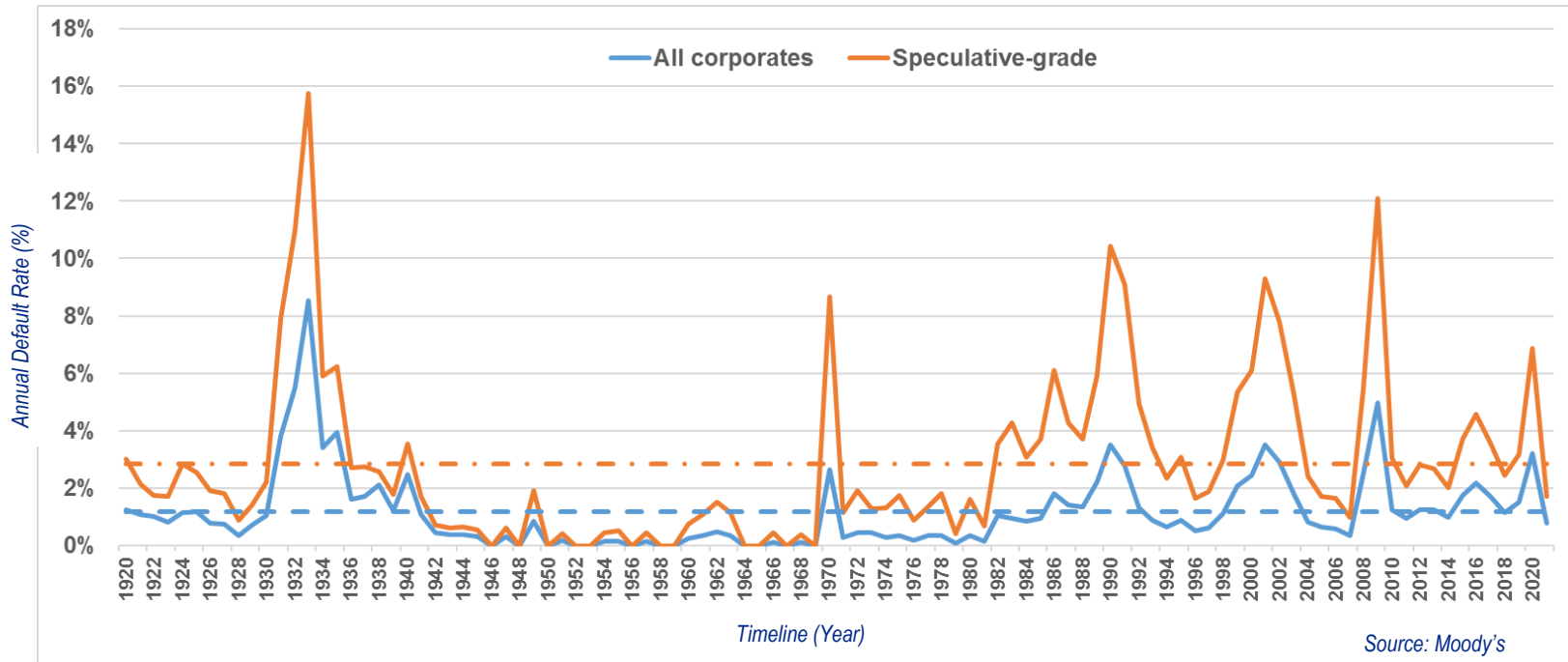
Credit risk in broader sense is also the risk of a decline in the credit quality of a counterparty. Such a deterioration does not imply default, but means that the probability of default increases (e.g., such that more difficult to sell the loan or bond; or that one needs to take more provisions or more Risk Weighted Assets). Credit quality is typically expressed by means of credit ratings

AAA, AA+, AA, AA-, A+, A, A-, BBB+, BBB, BBB-, BB+, BB, BB-, B+, B, B-, CCC, CC, C, D

Investment grade	Speculative grade	Default
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Default rates fluctuate on the credit quality and the economic conditions

Models capture default rate cyclicalty and differences of credit quality



- ❑ The **difference in credit quality** explains why the default risk on a speculative grade portfolio is higher than on a global portfolio with investment grade and speculative credits
- ❑ **Investment grade** default rates are nearly zero during expansion periods and peak during recessions
- ❑ **Speculative grade** default rates can triple or quadruple during recession periods
- ❑ **Models** are needed to **differentiate** the default risk and the capture the “**Point-in-Time**” **cyclicalty** of the default risk on the remaining lifetime of the credit and especially on the first years of the business cycle

It is important to differentiate well between high and low risk credits; and to calibrate the risk level well in line with its pricing

Transactions risk models differentiate between high and low risk

When granting a loan, one takes a credit risk. To determine the level of risk of a transaction, one segments transactions into pools of homogeneous risk. To each homogeneous risk bucket, one quantifies the risk level in the calibration step.

Discrimination & segmentation: the more precise the risk measurement, the better :

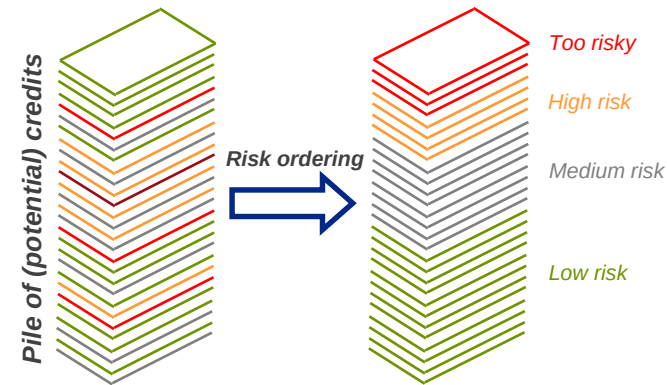
- To which counterparts should the bank give a loan to?
- Can one discriminate between good counterparts (low risk) and bad counterparts (high risk)?
- The better the discrimination, the better one can avoid bad customers or can one take preventive actions for existing customers evolving negatively.

Calibration:

- For the given homogeneous group, can we measure the risk level accurately?
- Which is the data history available for the calibration?
- Can we make sure that the average risk level will not change in the next years? What is the impact of the business cycle.

Applied techniques:

- Mathematical engineering techniques – statistics – econometrics – data science – computational finance
- Knowledge fusion of quantitative engineers with financial experts
- Development rules imposed by local and international regulation (gender, data history, statistical significance, transparency, ...)



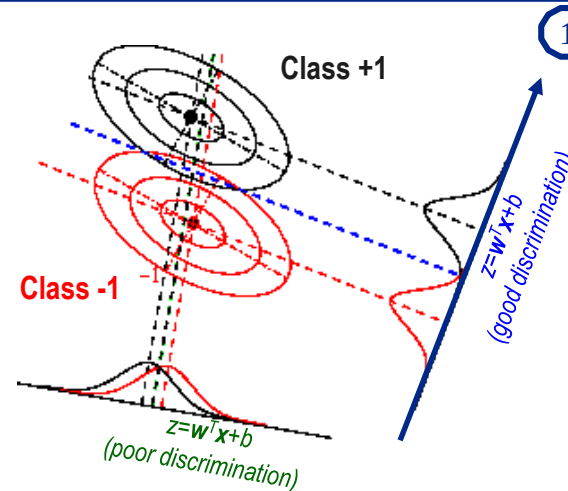
= $\text{xx}^?$ € **Cost of Risk**

**Evolution over time
In base/stress case**

A model development implies a discrimination, segmentation and calibration technique

General structure for a PD rating model (standalone rating, without support / country risk)

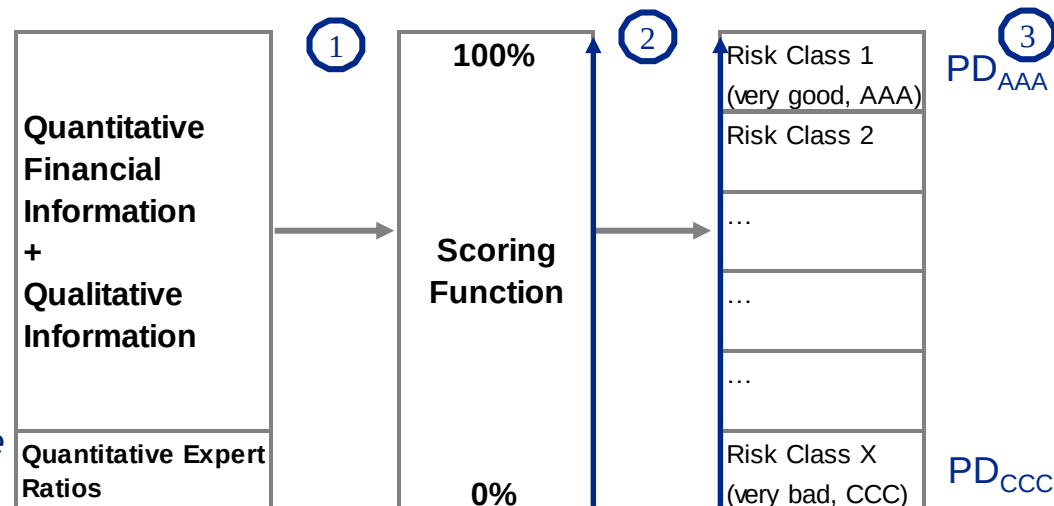
- ① Discrimination step: The financial strength score is obtained as a weighted combination of explanatory variables
- Financial ratios are e.g., leverage, profitability, capital adequacy, ...
 - Qualitative objective factors are e.g., industry sector, region;
 - Qualitative judgmental factors are determined internally, e.g., management experience/quality



- ① Discrimination models aim to optimally separate the good from the bad class. Well known discrimination techniques are, a.o., Linear Discriminant Analysis (LDA (illustrated in the left graph) OLS, logit), additive models, (deep) neural nets & kernel learning, (forests of) trees and rules,

- ② Segmentation step: The financial strength score is segmented into score segments or financial strength ratings
- Within each segment/rating, the default risk is uniform
 - The default risk differs between risk classes
 - Segmentation catalyses stable rating migrations

- ③ Calibration step: A conservative/best estimate PD that is Through-The-Cycle or Point-in-Time is assigned to each rating class



The main risk parameters for an individual credit are the Probability of Default (PD), Loss Given Default (LGD) and Exposure at Default (EAD)

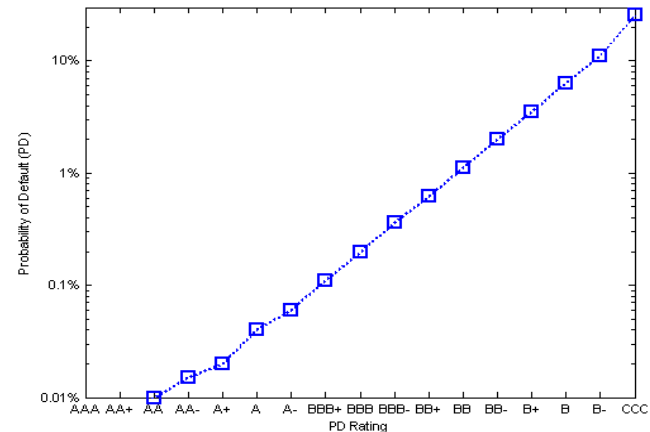
Credit risk models are needed for Default (PD), loss (LGD) & exposure risk (EAD)

Basel and EBA/ECB regulation catalyzed the uniformization of the building blocks for credit risk measurement of a transaction:

- Each loan may default: risk of default is expressed by a Probability of Default (PD) ·

The PD is expressed via the PD rating, which is typically expressed on a scale from AAA to CCC:

AAA (very low risk), AA, A, BBB, BB, B, CCC, CC, C (very high risk)



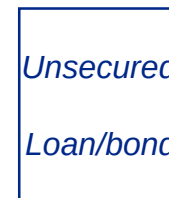
- Each defaulted loan represents a loss: Loss Given Default (LGD) ranging from 0 – 100%



Mortgage



Coll. loan



High LGD

- At time of default, the Exposure At Default (EAD) represents the outstanding amount. For a loan, the EaD is deterministic, for a credit facility (e.g., a credit card), the exposure at time of default is uncertain and needs to be measured.

The portfolio loss depends on the individual risk parameters (PD, LGD, EAD) and their interdependencies. Portfolio losses are fat-tailed.

Credit risk on a portfolio: 1-year view

- Given the default probability (PD), loss given default (LGD) and exposure at default (EAD) for every transaction, the portfolio loss distribution is given by*

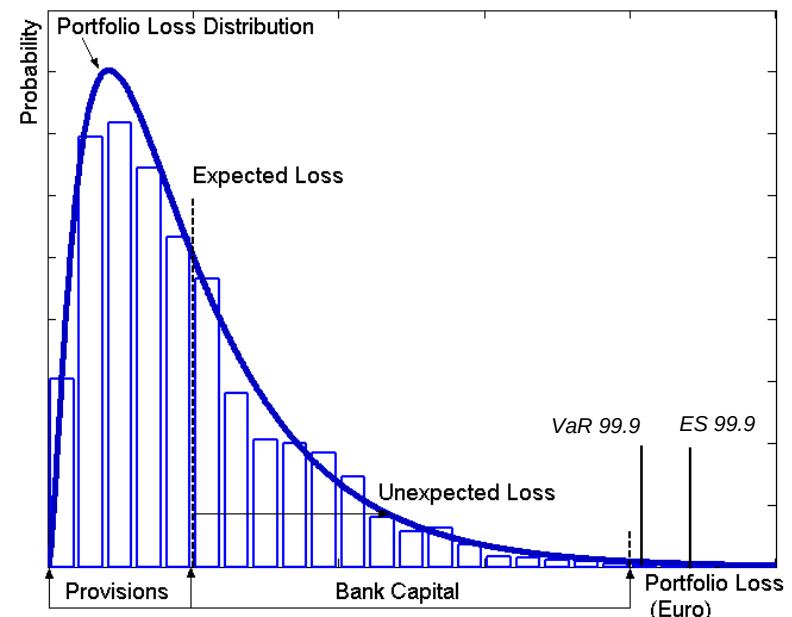
$$\text{Portfolio Loss} = \sum_i E\tilde{A}D_i \times L\tilde{G}D_i \times \mathbf{1}_{PD_i}$$

- The average outcome (Expected Loss)

$$\text{Expected Loss} = \sum_i E\bar{A}D_i \times L\bar{G}D_i \times PD_i$$

is compensated by the credit margin. The expected losses on a 1 and multi-year horizon are the basis for pricing, financial planning and provisioning.

- The uncertainty around the expected loss is typically a fat-tailed distribution:
 - Credit portfolio losses are fat-tailed due to the correlation/cyclical effects or due to concentration effects
 - The risk of large losses is expressed by the Value-at-Ris (VaR) and/or Expected Shortfall (ES).
 - Bank capital is required to cover unexpected losses. The more risky, the more capital is needed.
 - Capital serves to cover the risk on a one-year horizon at high confidence interval level: VaR 99.9% or 1 out of 100

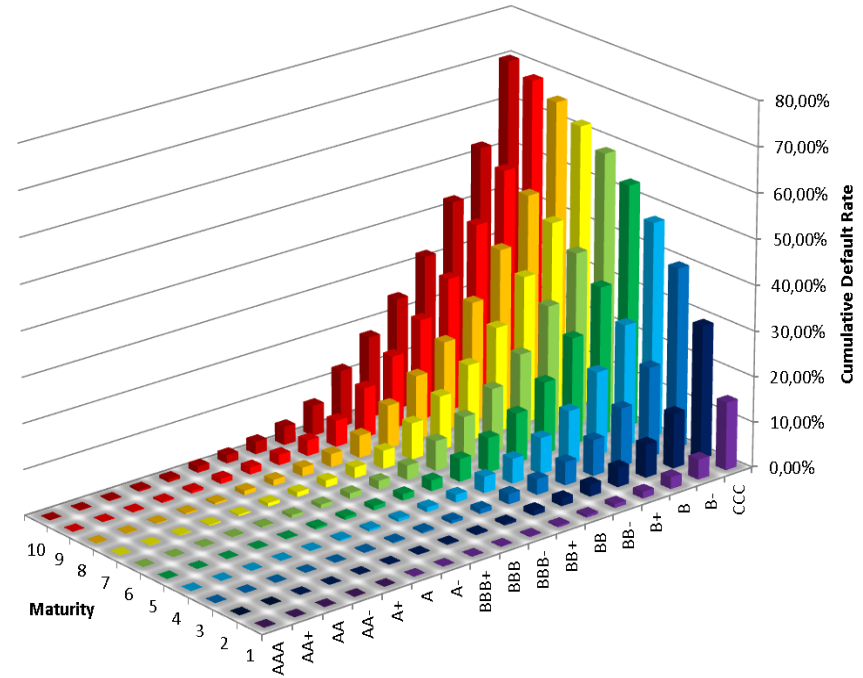


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Risk Parameters – Probability of Default by time horizon

Term structure of default risk

- ❑ The **1-year default probability** is used for capital calculation as defined by Basel II and Basel III
- ❑ Investments are made on a **longer time horizon**
 - High quality ratings are more likely to degrade with time: increasing default rate with time
 - Low quality ratings are more likely to upgrade with time (if not defaulted): reducing default rate with increasing maturity
- ❑ **Long-term default rates** are needed for valuation, investment decisions and provisioning (IFRS9 accounting rules for exposures in bucket II)

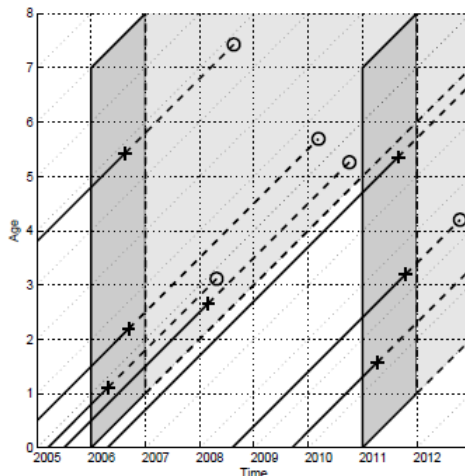


Remark: term structure is the most dominant for default risk; similar concepts apply to loss and exposure risk, possibly with deterministic trends (e.g., expected decrease of the LTV in amortizing mortgages). For loss and exposure risk, the main difference is due to the change of prediction horizon, not due the cumulative effect as for PD.

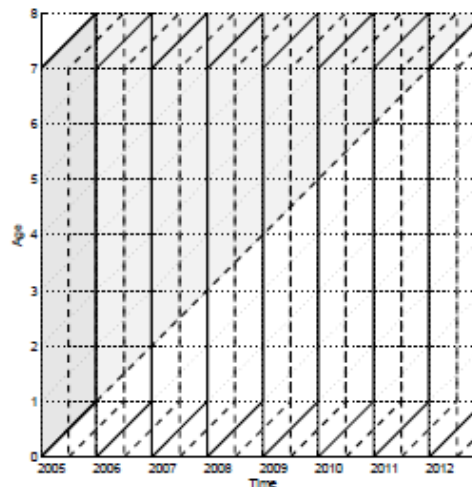
Risk Parameters – Probability of Default

Term structure of default risk: measurement

- Measurement of default risk visualized in the **Lexis chart** (x: Time, y: Age): each credit starts at the x-axis. Vertical lines define cohorts; horizontal segments define vintages. A line stops because of default (x) or (early) end of loan (o). The default rate for a **cohort** is measured by the number of loans (N) at starting date (crossing the vertical line), by the number of defaults (Nd) in the measurement tetragon and by the number of stopped /withdrawn (Nw) loans¹.



(a) Lexis chart



(b) Multiple cohorts

Uncorrected cohort DR: N_d/N

Withdrawal corrected dynamic cohort DR: $N_d/(N-N_w)$

Withdrawal corrected dynamic cohort DR: $N_d/(N-N_w/2)$
(random censoring)

Duration based: using time in the interval

- For **averages across cohorts**, one can choose overlapping or non-overlapping measurement periods. Note that there are more cohorts available to compute short term default rates than long term default rates.

Risk Parameters – Probability of Default

Term structure of default risk: measurement

- Default rates can be defined from 0 to T or for any period (t1 ... t2) in between. Such default rates are called marginal default rates. The default rate in year 2 of the 2000 cohort of 4.1% represents the percentage of defaulting firms defaulting during 2002 that that were given a rating/loan in 2000 and that did not default/withdraw in 2001.

Nobs	year	age												
		1	2	3	4	5	6	7	8	9	10	11	12	13
3845	2000	2.7	4.1	3.3	2.1	1.0	0.6	0.7	0.3	1.2	2.6	0.2	0.5	0.4
3842	2001	4.1	3.6	2.2	1.0	0.7	0.7	0.3	1.3	2.5	0.2	0.6	0.2	
3810	2002	3.4	2.2	1.0	0.6	0.6	0.2	1.4	2.5	0.3	0.5	0.5		
3743	2003	2.0	0.9	0.6	0.6	0.2	1.4	2.6	0.3	0.5	0.4			
3842	2004	0.9	0.6	0.6	0.3	1.2	3.0	0.4	0.4	0.5				
4031	2005	0.7	0.6	0.4	1.5	3.5	0.5	0.6	0.7					
4160	2006	0.6	0.4	1.9	3.9	0.8	0.8	0.8						
4306	2007	0.4	2.1	4.5	1.2	0.7	1.0							
4371	2008	2.1	5.0	1.4	0.8	1.0								
4236	2009	5.2	1.4	0.7	1.1									
3988	2010	1.3	0.8	1.1										
4243	2011	0.8	1.3											
4416	2012	1.2												

$\overline{DR}_{0,1}$ $\overline{DR}_{1,2}$ $\overline{DR}_{12,13}$

- The **multiyear average** is typically calculated based upon single year averages: using more data on short term and less on longer term: 13 measurement points for the first year and 1 for the last year (13)

$$\overline{DR}_{0,T} = 1 - \prod_{t=1}^T (1 - \overline{DR}_{t-1,t}).$$

Risk Parameters – Probability of Default

Term structure of default risk: model reasons

- **Empirical multiyear default rates** need idealization because of sample noise (example below based upon default rates reported by external agencies Moody's, S&P, Fitch).

Maturity		AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC
	1	0,00%	0,00%	0,00%	0,00%	0,00%	0,06%	0,05%	0,15%	0,11%	0,24%	0,31%	0,32%	1,00%	1,46%	3,17%	4,32%	16,17%
	5	0,00%	0,00%	0,00%	0,00%	0,00%	0,62%	0,65%	0,54%	1,84%	2,44%	5,01%	5,77%	12,21%	16,86%	23,83%	30,13%	51,41%
	10	0,00%	0,00%	0,00%	0,00%	0,24%	1,74%	2,02%	1,17%	3,76%	6,83%	12,62%	14,27%	25,87%	34,04%	40,50%	50,59%	66,92%
	15	0,00%	0,00%	0,00%	0,00%	1,03%	4,34%	3,50%	3,85%	6,32%	12,41%	21,48%	29,05%	37,09%	47,38%	45,61%	58,13%	83,27%

- Empirical data may also be too short term data history and extrapolation is needed to estimate full lifetime loss
- **Smoothness conditions** are expected by financial experts, though may not be observed, a.o., due to small sample noise.
- It is also convenient to have the default term structure by rating into tractable form to mix short term and long term calculations
- The term structure differs by asset classes depending a.o. on their rating stability.

Risk Parameters – Probability of Default

Model formulation: Homogeneous Continuous Time Markov Chain

The idealization of the cumulative default rates is done using a **generator migration** matrix :

- Single year migration matrix: $M(1 \text{ year}) = \exp(Q.1)$
- Multiyear migration matrix: $M(t \text{ year}) = M^t = \exp(Q.t)$
- The t -year cumulative default rates $DR(t, Q)$ are obtained as the last column of $M(t \text{ year})$
- The Markov chain is defined by the generator matrix Q of the continuous time Markov chain; or its discrete time equivalent $M(1 \text{ year})$.

YY+1	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	86.82%	6.41%	2.90%	1.44%	1.42%	0.15%	0.14%	0.12%	0.11%	0.10%	0.09%	0.08%	0.06%	0.05%	0.04%	0.03%	0.02%	0.01%
AA+	1.43%	88.10%	2.91%	2.87%	1.48%	1.46%	0.55%	0.54%	0.17%	0.10%	0.09%	0.08%	0.07%	0.05%	0.04%	0.03%	0.02%	0.01%
AA	0.13%	2.91%	74.91%	8.04%	7.84%	3.54%	1.61%	0.27%	0.26%	0.10%	0.09%	0.08%	0.07%	0.06%	0.05%	0.03%	0.02%	0.01%
AA-	0.01%	0.63%	2.72%	73.14%	16.33%	5.35%	1.01%	0.15%	0.13%	0.10%	0.09%	0.08%	0.07%	0.06%	0.05%	0.04%	0.03%	0.02%
A+	0.01%	0.01%	0.12%	0.83%	76.29%	15.57%	5.52%	0.65%	0.40%	0.11%	0.10%	0.09%	0.08%	0.07%	0.06%	0.05%	0.04%	0.02%
A	0.01%	0.01%	0.16%	0.45%	0.93%	73.23%	17.95%	4.43%	1.63%	0.35%	0.34%	0.11%	0.10%	0.08%	0.07%	0.06%	0.05%	0.04%
A-	0.01%	0.01%	0.02%	0.02%	0.35%	1.72%	78.72%	13.29%	4.29%	0.51%	0.24%	0.23%	0.16%	0.11%	0.10%	0.09%	0.08%	0.07%
BBB+	0.00%	0.01%	0.01%	0.02%	0.02%	0.30%	3.33%	77.02%	13.33%	3.18%	0.57%	0.56%	0.55%	0.53%	0.16%	0.15%	0.14%	0.12%
BBB	0.00%	0.01%	0.01%	0.01%	0.06%	0.12%	0.66%	2.58%	74.84%	16.30%	1.76%	1.00%	0.93%	0.65%	0.59%	0.18%	0.16%	0.15%
BBB-	0.00%	0.01%	0.01%	0.03%	0.03%	0.22%	0.45%	0.77%	4.86%	75.59%	9.04%	3.30%	2.23%	0.95%	0.77%	0.75%	0.74%	0.25%
BB+	0.01%	0.01%	0.01%	0.02%	0.02%	0.02%	0.03%	0.03%	0.91%	4.71%	70.51%	8.26%	7.70%	5.27%	0.70%	0.68%	0.66%	0.46%
BB	0.00%	0.01%	0.01%	0.01%	0.01%	0.06%	0.07%	0.09%	0.43%	1.66%	3.74%	68.48%	14.40%	4.04%	3.87%	1.95%	0.59%	0.57%
BB-	0.00%	0.01%	0.01%	0.01%	0.06%	0.11%	0.11%	0.11%	0.12%	0.17%	0.74%	4.25%	64.72%	16.89%	4.60%	4.45%	2.17%	1.47%
B+	0.00%	0.01%	0.01%	0.01%	0.02%	0.02%	0.02%	0.03%	0.03%	0.09%	0.09%	0.91%	3.84%	65.97%	16.77%	6.45%	4.15%	1.59%
B	0.00%	0.01%	0.01%	0.01%	0.01%	0.02%	0.02%	0.02%	0.03%	0.03%	0.07%	0.30%	0.75%	4.66%	54.80%	17.84%	17.34%	4.08%
B-	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.02%	0.02%	0.02%	0.17%	0.37%	2.80%	61.19%	25.79%	9.54%	
CCC	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.02%	0.02%	0.02%	0.04%	0.04%	0.04%	0.13%	0.48%	1.48%	73.57%	24.12%

- The Markov assumption assumes that all present information is summarized in the rating, which is contradictory to reported¹ **rating downgrade momentum** and **rating stickiness** ('duration dependence effect'). The migration matrix can be non-homogeneous. Note that depending on the definition and sector, D may or may not be an absorbing state.

Through-The-Cycle Term Structure of Default Risk

Model Formulation: Non-Homogeneous Continuous Time Markov Chain

- Relaxation of the homogenous continuous time Markov property, the risk level develops differently in time depending on the rating*:
- On **short term**: non-homogeneous continuous time Markov chain ($t \rightarrow \varphi_{\alpha\beta}(t)$):

$$M(t \text{ year}) = \exp(Q \cdot \varphi_{\alpha\beta}(t))$$

with

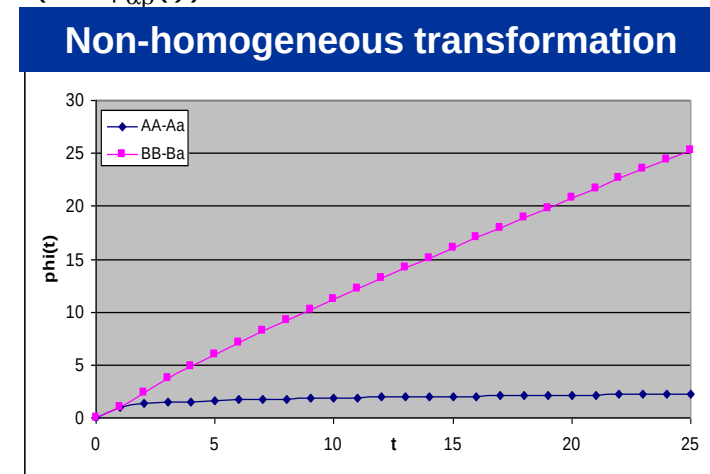
$\varphi_{\alpha\beta}(t)$ a **diagonal matrix with diagonal elements**

$$\varphi_{\alpha\beta}(t) = t \times (1 - \exp(-\alpha t)) t^{\beta-1} / (1 - \exp(-\alpha))$$

- On **long term**: homogeneous continuous time Markov chain:

$$M(t \text{ year}) = \exp(Q \cdot t)$$

- The homogeneous continuous time Markov chain on longer term periods limits extrapolation problems. A **smooth transition between non-homogeneous and homogeneous** time is obtained by a dedicated interpolation to ensure that $\varphi_{\alpha\beta}(t)$ increases with time
- The **parameters Q, α, β are estimated from empirical data**. The increased flexibility of the Markov chain improves the fit on short term.



Through-The-Cycle Term Structure of Default Risk

Simple model estimation techniques often lack precision

The elements of the matrix Q are estimated from*

❑ **Observed migration matrices** $Q = \log(M(1 \text{ year observed}))$

- + Simple approach, single year migration matrices can be averaged; easily available, single year migration matrices are the reference for many applications (stress test, value at risk, pricing) ; well distributed sample of expansion and recession periods
- Small sample noise on off diagonal elements, important extrapolation when using 1-year averages. One can also use multiyear averages, with risk of non-stochastic matrix and interpolation/extrapolation

❑ Estimated on observed cumulative default rates $DR(t, \text{obs})$ via **sum squared error**

$$\sum_{AAA-CCC} \sum_{t=1}^T (DR(t, \text{obs}) - DR(t, Q))^2$$

- + Extensive use of empirical data on longer time horizon, least squares is a standard problem formulation
- Complex optimization; introduction of weights? To deal with small sample noise and to deal with large range of default rates (AAA – CCC) and time dimension makes absolute errors difficult

Through-The-Cycle Term Structure of Default Risk

Refined model estimation techniques with complexity trade-off

- **Survival analysis likelihood:** the likelihood L_i of an observation depends on the observed outcome over the observation period of T years:

- Default event in year t of the observation period: $L_i = PD(t) - PD(t - 1)$

- Rating withdrawal in year t of the observation period: $L_i = 1 - PD(t - 1)$

- No default/withdrawal till the end of the observation period: $L_i = 1 - PD(T)$

* $PD(k)$ is the probability to default within k years; in the Markov chain framework it is in the last column of the multiyear migration matrix.

- The full likelihood is the product of all the individual likelihoods over all observations and cohorts:

$$L = \prod_i L_i$$

- From the log-likelihood complexity criteria can be derived, e.g. a **Bayesian Information Criterion (BIC)** can be defined as follows (with n the number of observations *):

$$BIC = -2 \log(L) + \text{number of parameters} \times \log(n)$$

The BIC can be used to compare different models with different parametrizations. It avoids overfitting by penalizing models with many free parameters.

* In the context of survival analysis with censoring this requires some care . Defaults account for 1 full observation but none-defaults only contain partial information: they should be weighted by the fitted probability of default until the time of withdrawal (or until the end of the observation period)¹.

Through-The-Cycle Term Structure of Default Risk

Ensuring applicability by business constraints and parameterization

Business constraints:

- ❑ Correct long term PDs when backtesting with observed DRs (confidence interval)
- ❑ Correct long term migrations when backtesting with observed migration probabilities (confidence interval)
- ❑ Smooth term structure
- ❑ Central tendency to the sectors average marginal default rate (no discrimination after many years)
- ❑ Intuitive behavior across asset classes
- ❑ Consistent migration matrix¹: increasing risk levels per rating to retain the risk ordering of ratings

$$\sum_{i=l}^q M(k, i) \leq \sum_{i=l}^q M(k+1, i), \quad k = 1 \dots q-1; l = 1 \dots q$$

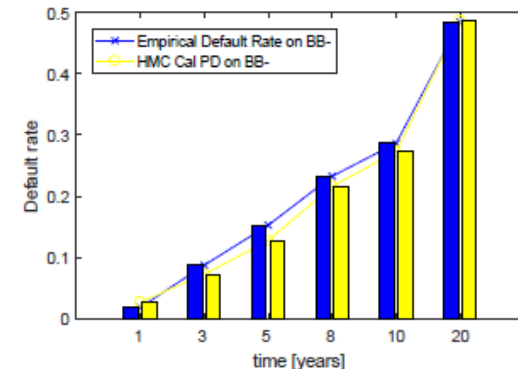
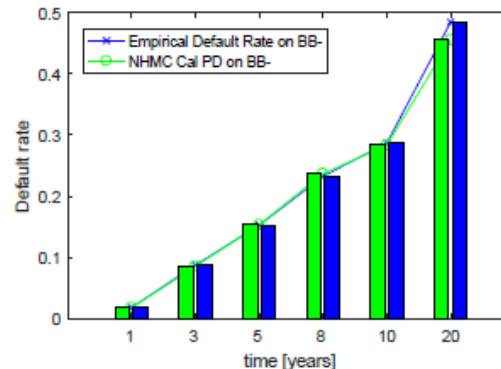
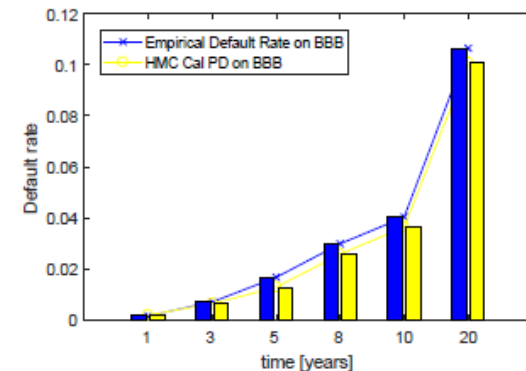
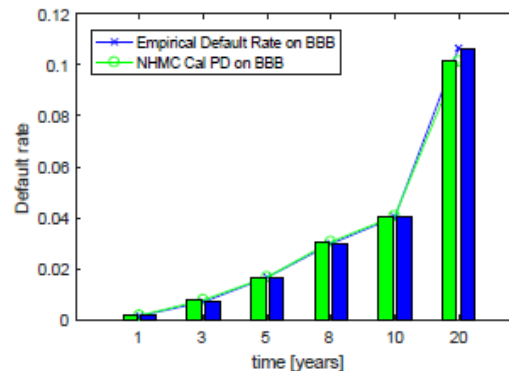
Parameterization:

- ❑ Starting point: empirical migration matrix
- ❑ Question how to parameterize the (full) migration matrix: each individual element, upper/diagonal/lower part; curse of dimensionality?
- ❑ The parameterization complexity is gradually increased in a semi-automated process

Through-The-Cycle Term Structure

Already graphically a refined model shows its benefits

- ❑ Comparison of homogeneous time and non-homogeneous time with empirical (withdrawn corrected) cumulative default rates
- ❑ Non-homogeneous continuous time Markov chain models improve the fit, especially on the lowest and highest ratings
- ❑ Design of a non-homogeneous continuous time model is more complex
- ❑ Formal **backtest** is reported at the end

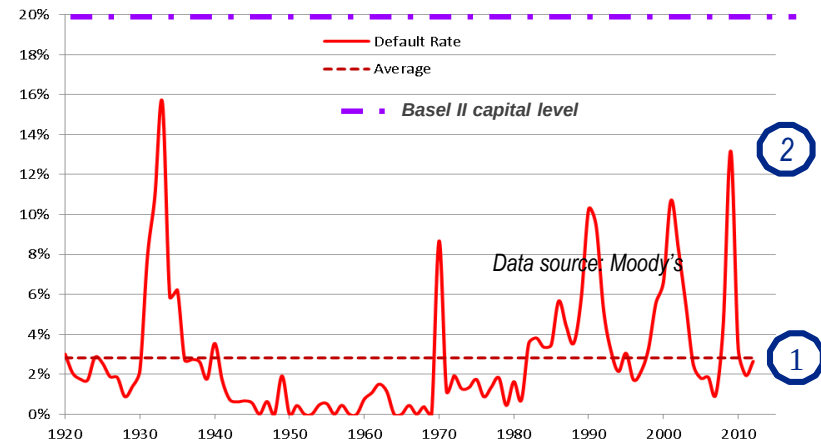


- ❑ Bank capital adequacy
- ❑ Credit risk models and parameters
- ❑ Multi-period models
- ❑ **Macro-economic panel data models**
- ❑ Implementation and application

Point-in-Time behavior: impact of the macro-economic cycle on top of the Through-the-Cycle behavior

Different definitions for the Probability of Default

- ❑ Ratings, rating systems and risk parameters can be Through-The-Cycle, Point-In-Time or a mixture of both.
- ❑ A **rating system** is **Through-The-Cycle** when the ratings are assigned with a risk view during the whole economic cycle to come. A **Point-in-Time ratings system** incorporates the current state of economy and its outlook.
- ❑ The PD risk parameter is Through-The-Cycle when it is constant for a given rating. The Point-In-Time PD fluctuates with the economic cycle.
- ① The Basel regulatory capital is based upon **Through-The-Cycle (conservative) average one-year** PD. The procyclicality argument favors TTC ratings and PDs.
- ❑ Point-in-time models take into account the
- ② common correlations between economic factors and default rates
- ❑ For stress tests, provisions, valuation, financial plan: migrations and default rates depend on the **point in time** within the business cycle. The Point-in-Time model adds the cyclical component to the best estimate Through-The-Cycle PD.



Point-in-Time PD: macro-economic factor model

Univariate & matrix formulation

Default risk levels fluctuate with the cycle (migrations and loss rates to a less extent)

- For the financial plan, point-in-time default rates are applied to reflecting economic downturn situation and stress scenarios
- Given the macro-economic scenario defined (internally or by the EC, ECB, EBA) the point-in-time effect is calculated via time-series models, e.g.,

$$DR_t = 0.002x + 0.4xxx \times DR_{t-1} - 0.001x \times \Delta_q GDP_t$$

Default rate
Default rate (1 quarterlag)
Quarterly GDP growth

- The loss rates are less dependent on the economic cycle:
- Exposure risk (CCF) is reported to be countercyclical*
- The point-in-time global default rate fluctuations are translated into impacts per rating scale and migrations. Migration impacts** are typically less severe than default rate fluctuations for corporates; though the inverse is observed for banks.

$$LGD_{PIT,t} = LGD_{AV,t} + \alpha + \beta \Delta GDP_t + \dots$$

PITmacroeconomic model

average

YY+1	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	86.82%	6.41%	2.90%	1.44%	1.42%	0.15%	0.14%	0.12%	0.11%	0.10%	0.09%	0.08%	0.06%	0.05%	0.04%	0.03%	0.02%	0.01%
AA+	1.43%	88.10%	2.91%	2.87%	1.48%	1.46%	0.55%	0.54%	0.17%	0.10%	0.09%	0.08%	0.07%	0.05%	0.04%	0.03%	0.02%	0.01%
AA	0.13%	2.91%	74.91%	8.04%	7.84%	3.54%	1.61%	0.27%	0.26%	0.10%	0.09%	0.08%	0.07%	0.06%	0.05%	0.03%	0.02%	0.01%
AA-	0.01%	0.63%	2.72%	73.14%	16.33%	5.35%	1.01%	0.15%	0.13%	0.10%	0.09%	0.08%	0.07%	0.06%	0.05%	0.04%	0.03%	0.02%
A+	0.01%	0.01%	0.12%	0.83%	76.29%	15.57%	5.52%	0.65%	0.40%	0.11%	0.10%	0.09%	0.08%	0.07%	0.06%	0.05%	0.04%	0.02%
A	0.01%	0.01%	0.16%	0.45%	0.93%	73.23%	17.95%	4.43%	1.63%	0.35%	0.34%	0.11%	0.10%	0.08%	0.07%	0.06%	0.05%	0.04%
A-	0.01%	0.01%	0.02%	0.02%	0.35%	1.72%	76.72%	13.29%	4.29%	0.51%	0.24%	0.23%	0.16%	0.11%	0.10%	0.09%	0.08%	0.07%
BBB+	0.00%	0.01%	0.01%	0.02%	0.02%	0.30%	3.33%	77.02%	13.33%	3.18%	0.57%	0.56%	0.55%	0.53%	0.16%	0.15%	0.14%	0.12%
BBB	0.00%	0.01%	0.01%	0.01%	0.06%	0.12%	0.66%	2.58%	74.84%	16.30%	1.76%	1.00%	0.93%	0.65%	0.59%	0.18%	0.16%	0.15%
BBB-	0.00%	0.01%	0.01%	0.03%	0.03%	0.22%	0.45%	0.77%	4.86%	75.59%	9.04%	3.30%	2.23%	0.95%	0.77%	0.75%	0.74%	0.25%
BB+	0.01%	0.01%	0.01%	0.02%	0.02%	0.03%	0.03%	0.91%	4.71%	70.51%	8.26%	7.70%	5.27%	0.70%	0.68%	0.66%	0.46%	0.46%
BB	0.00%	0.01%	0.01%	0.01%	0.01%	0.06%	0.07%	0.09%	0.43%	1.66%	3.74%	68.48%	14.40%	4.04%	3.87%	1.95%	0.59%	0.57%
BB-	0.00%	0.01%	0.01%	0.01%	0.06%	0.11%	0.11%	0.12%	0.17%	0.74%	4.25%	64.72%	16.89%	4.80%	4.45%	2.17%	1.47%	1.47%
B+	0.00%	0.01%	0.01%	0.01%	0.02%	0.02%	0.03%	0.03%	0.09%	0.09%	0.91%	3.84%	65.97%	16.77%	6.45%	4.15%	1.59%	1.59%
B	0.00%	0.01%	0.01%	0.01%	0.01%	0.02%	0.02%	0.03%	0.03%	0.07%	0.30%	0.75%	4.66%	54.80%	17.84%	17.34%	4.08%	4.08%
B-	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.02%	0.02%	0.17%	0.37%	2.80%	61.19%	25.79%	9.54%	9.54%	9.54%
CCC	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.02%	0.02%	0.02%	0.04%	0.04%	0.04%	0.13%	0.48%	1.48%	73.57%	24.12%

downturn

YY+1	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC	D
AAA	84.14%	7.59%	3.52%	1.77%	1.75%	0.18%	0.17%	0.15%	0.14%	0.12%	0.11%	0.10%	0.08%	0.07%	0.05%	0.04%	0.02%	0.01%
AA+	1.15%	86.17%	3.46%	3.46%	1.80%	1.79%	0.68%	0.67%	0.21%	0.12%	0.11%	0.10%	0.08%	0.07%	0.05%	0.04%	0.02%	0.01%
AA	0.10%	2.36%	71.54%	9.17%	9.27%	4.31%	1.98%	0.33%	0.32%	0.13%	0.11%	0.10%	0.08%	0.07%	0.06%	0.04%	0.03%	0.02%
AA-	0.00%	0.51%	2.21%	69.66%	18.86%	6.49%	1.25%	0.18%	0.17%	0.13%	0.11%	0.10%	0.09%	0.07%	0.06%	0.05%	0.03%	0.02%
A+	0.00%	0.01%	0.10%	0.67%	72.46%	18.01%	6.71%	0.80%	0.49%	0.13%	0.12%	0.11%	0.09%	0.08%	0.07%	0.06%	0.04%	0.03%
A	0.00%	0.01%	0.12%	0.34%	0.70%	67.91%	21.50%	5.69%	2.13%	0.46%	0.45%	0.14%	0.13%	0.11%	0.10%	0.08%	0.07%	0.05%
A-	0.00%	0.01%	0.01%	0.02%	0.27%	1.31%	74.46%	16.29%	5.56%	0.67%	0.31%	0.30%	0.21%	0.15%	0.13%	0.12%	0.10%	0.09%
BBB+	0.00%	0.01%	0.01%	0.01%	0.19%	2.17%	70.53%	18.11%	4.71%	0.87%	0.85%	0.83%	0.82%	0.25%	0.23%	0.21%	0.19%	0.19%
BBB	0.00%	0.00%	0.01%	0.01%	0.04%	0.08%	0.43%	1.72%	68.00%	21.69%	2.56%	1.47%	1.39%	0.97%	0.89%	0.27%	0.25%	0.23%
BBB-	0.00%	0.00%	0.01%	0.02%	0.02%	0.14%	0.30%	0.51%	3.28%	70.61%	12.03%	4.66%	3.24%	1.40%	1.15%	1.13%	1.11%	0.38%
BB+	0.00%	0.01%	0.01%	0.01%	0.01%	0.02%	0.02%	0.60%	3.16%	63.97%	10.36%	10.43%	7.62%	1.04%	1.02%	1.00%	0.70%	0.70%
BB	0.00%	0.00%	0.01%	0.01%	0.04%	0.04%	0.06%	0.29%	1.10%	2.53%	61.72%	18.31%	5.61%	5.59%	2.91%	0.89%	0.87%	0.87%
BB-	0.00%	0.00%	0.01%	0.01%	0.04%	0.07%	0.07%	0.08%	0.08%	0.11%	0.49%	2.86%	57.15%	20.90%	6.31%	6.39%	3.22%	2.22%
B+	0.00%	0.00%	0.01%	0.01%	0.01%	0.01%	0.02%	0.02%	0.06%	0.06%	0.60%	2.57%	58.29%	20.87%	8.97%	6.09%	2.40%	2.40%
B	0.00%	0.00%	0.01%	0.01%	0.01%	0.01%	0.02%	0.02%	0.02%	0.04%	0.20%	0.49%	3.14%	46.39%	20.28%	23.27%	6.08%	6.08%
B-	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.10%	0.23%	1.75%	50.73%	32.44%	14.67%	14.67%	14.67%
CCC	0.00%	0.00%	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.02%	0.02%	0.02%	0.08%	0.29%	0.92%	64.44%	34.13%

Point-in-Time PD: macro-economic + latent factor model

Panel data structure – rating & time series dimension

Rating Period	AAA Aaa	AA Aa	A A	BBB Baa	BB Ba	B B	CCC/C Caa-C
1982	0.00	0.00	0.19	0.28	3.00	2.28	18.74
1983	0.00	0.00	0.00	0.16	0.79	3.78	13.19
1984	0.00	0.00	0.00	0.42	0.78	3.42	37.80
1985	0.00	0.00	0.00	0.00	1.02	4.73	7.69
1986	0.00	0.00	0.09	0.42	1.17	6.99	16.97
1987	0.00	0.00	0.00	0.00	0.69	2.72	9.76
1988	0.00	0.00	0.00	0.00	0.87	3.50	17.65
1989	0.00	0.18	0.00	0.48	1.24	4.10	23.72
1990	0.00	0.00	0.00	0.29	2.85	9.18	34.52
1991	0.00	0.00	0.00	0.37	2.42	10.77	31.19
1992	0.00	0.00	0.00	0.00	0.08	5.82	22.60
1993	0.00	0.00	0.00	0.00	0.54	2.76	15.33
1994	0.00	0.00	0.07	0.00	0.20	2.49	9.59
1995	0.00	0.00	0.00	0.09	0.70	3.43	16.41
1996	0.00	0.00	0.00	0.00	0.34	1.72	4.75
1997	0.00	0.00	0.00	0.13	0.14	2.19	9.30
1998	0.00	0.00	0.00	0.25	0.67	3.51	24.55
1999	0.00	0.09	0.09	0.14	0.90	5.20	22.39
2000	0.00	0.00	0.13	0.35	0.93	6.29	24.53
2001	0.00	0.00	0.21	0.25	2.16	9.52	35.52
2002	0.00	0.00	0.07	0.87	1.95	5.69	32.55
2003	0.00	0.00	0.00	0.12	0.59	2.64	23.10
2004	0.00	0.00	0.04	0.00	0.32	0.96	10.78
2005	0.00	0.00	0.00	0.08	0.16	1.10	6.06
2006	0.00	0.00	0.00	0.00	0.20	0.68	7.60
2007	0.00	0.00	0.00	0.00	0.10	0.13	8.88
2008	0.00	0.43	0.37	0.41	0.80	2.72	18.24
2009	0.00	0.00	0.18	0.56	1.08	7.62	35.30
2010	0.00	0.00	0.05	0.00	0.28	0.53	14.18
2011	0.00	0.00	0.00	0.06	0.06	0.80	10.44
2012	0.00	0.00	0.00	0.02	0.19	0.89	16.33

Example of static pool one-year default rate statistics per broad rating on a wholesale portfolio: for each rating (column), one reports the one-year default rate per calendar year (rows). Default rates are nearly zero in the AAA zone and rise almost exponentially to 15-30% at CCC. For the investment grade ratings, default rates are almost zero during expansion periods

Point-in-Time model formulation:

Panel data structure: impacts and dependencies by rating:

- Univariate time series → multiple output model
- logit/probit with threshold/bias per rating
- constant or different macro-economic dependencies per rating (e.g., AAA in this example)
- Stationarity tests
- non-linear impacts? Cf multi-scenario in IFRS9
- common errors?

Point-in-Time PD: macro-economic + latent factor model

Generalized Linear Mixed Model with probit link function

The migration matrix is split in two parts that are modelled separately: one model for default and one model for the none-default migrations.

1. Default Model:

$$PD_{k,t} = \Phi(\theta_{r(k)} + X_t\beta + Z_t)$$

- Φ is the standard normal cumulative distribution function.
- $r(k)$ is the rating of obligor k .
- $\theta_{r(k)}$ is an intercept for rating $r(k)$
- X_t is an $1 \times p$ vector containing relevant macro-economic variables.
- β is a $p \times 1$ vector with the coefficients modeling the impact of the macro economic variables on the PD.
- $Z_t \sim N(0, \sigma^2)$ a latent factor with variance σ^2 . Latent factors can be correlated over time with correlation matrix $Corr(Z_1, \dots, Z_T) = C$.

2. Migration Model:

$$\begin{aligned} P_{down,k,t} &= \Phi(\theta'_{r(k)} + X'_t\beta' + Z'_t) \\ P_{stable,k,t} &= 1 - P_{down,k,t} - P_{up,k,t} \\ P_{up,k,t} &= \Phi(\theta''_{r(k)} + X'_t\beta' + Z'_t) \end{aligned}$$

- Complex migration probabilities are reduced to down, up and stable. Optionally, one can specify also up to 1, 2, 3 notch migrations
- Same formulation as default model, different parameters
- Further model complexities apply rating dependent coefficients (e.g., limited PiT dependency for AAA) and nonlinear transformations (additive model)

- *Latent factor allows incorporating correlations: Intra year correlations « state of the economy » in one year that affects all the ratings.*
- *Autoregressive Variables incorporate Extra-year correlations: If last year was a bad year and many defaults occurred, probably the default rates will also increase the following year.*
- *Macroeconomic Variables incorporate a fixed effect related to a specific indicator.*
- *Probit link function means equivalency with a structural Merton-type model with moving thresholds*
- *Parameters can be estimated by Maximum Likelihood with a Gauss-Hermite approximation of the likelihood (implementation glmer in R).*
- *Classical likelihood inference is readily available: significance testing on coefficients (p-values), BIC, ...*

The bias-variance trade-off in model selection also embeds a performance-readability trade-off

Refined model estimation techniques with complexity trade-off

Model 1: estimate a linear model* to determine the score z as a function of ratios $x_1, x_2, \dots, x_n$

$$Z = w_1x_1 + w_2x_2 + \dots + w_nx_n$$

Model 2: estimate nonlinear transformations f_i for some of the ratios (additive models)

$$Z = w_1x_1 + w_2f_2(x_2) + \dots + w_nf_n(x_n)$$

Model 3: estimate additional SVM terms

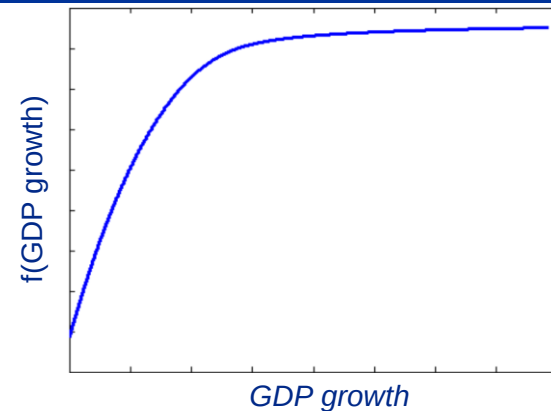
$$Z = w_1x_1 + w_2f(x_2) + \dots + w_nf(x_n) + f_{\text{SVM}}(x_2, x_n)$$

↑
Readability
Performance
↓

Complexity trade-off

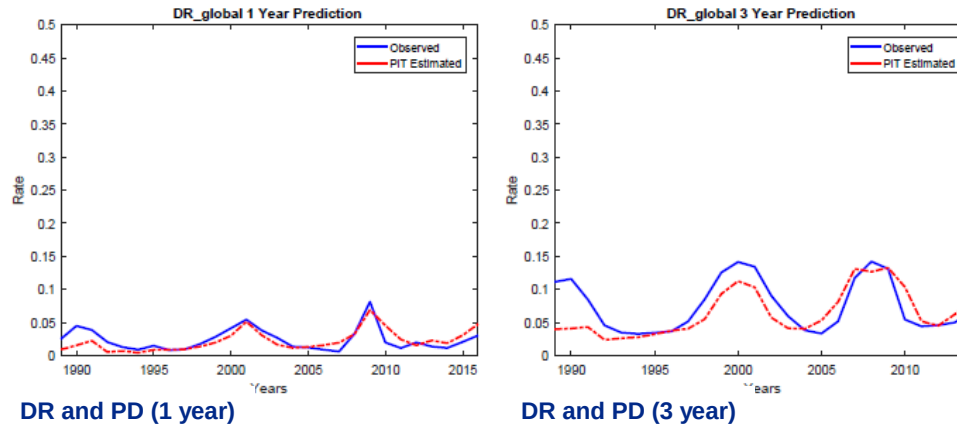
- Significance testing
- Complexity criteria (AIC, BIC)
- Out of sample testing or cross-validation
- Techniques to explain/interpret complex models
- **Model risk increases with model complexity, model governance and model risk control are key attention points given their importance**

Illustration of a nonlinear transformation



Point-in-Time PD: macro-economic + latent factor model

Graphical Performance



Graphical model performance evaluation:

- Earlier years weight less as they represent less data (an implicit forgetting factor), which results into lower performance
- Default cyclicalty is well captured, especially for the recent downturn periods

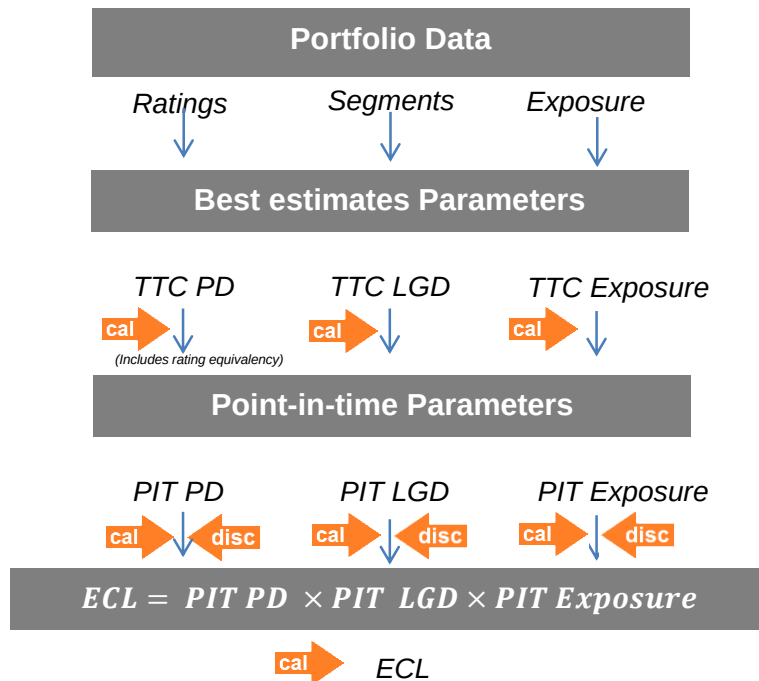
Final model selection is based upon statistical criteria such as log likelihood, BIC, out of sample performance (when possible) and financial expertise.

A formal backtest can be done to limit model risk before its application.

Note: the accuracy of Point-in-Time projections for the base case view depends also on the accuracy of the base case macro-economic scenario

Backtesting portfolio model parameters and predictions

A detailed test scheme of individual models and the global implementation is required



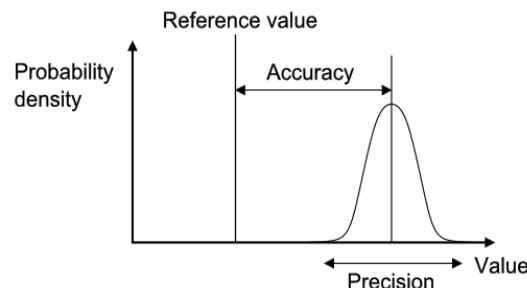
The backtest is concerned with the TTC parameters, the PIT parameters and the global result.

- Calibration test (cal): to check the accuracy.
- Discrimination test (disc): to check the precision, differentiation between high and low risk.
- Stability test: to check population stability to ensure risk parameters are developed on a representative reference data sets.

A backtest scheme priorities the critical tests; other tests can be put as optional in the procedures.

The key assessment for IFRS9 provisions is the calibration test that checks on average that the observed PD/LGD are in-line with model PD/LGD.

The backtests measure the difference between the observed values and the expected reference value. The results are presented in terms of a colour code (traffic lights approach). In contrast to backtesting regulatory parameters, too conservative results should also be remediated.



p-values		color code		interpretation
0% ≤ p <	5%	blue		conservative
5% ≤ p <	10%	cyan		weakly conservative
10% ≤ p <	50%	green		in line
50% ≤ p <	90%	yellow		in line
90% ≤ p <	95%	amber		weakly optimistic
95% ≤ p ≤	100%	red		optimistic.

- ❑ Bank capital adequacy
- ❑ Credit risk models and parameters
- ❑ Multi-period models
- ❑ Macro-economic panel data models
- ❑ **Implementation and application**

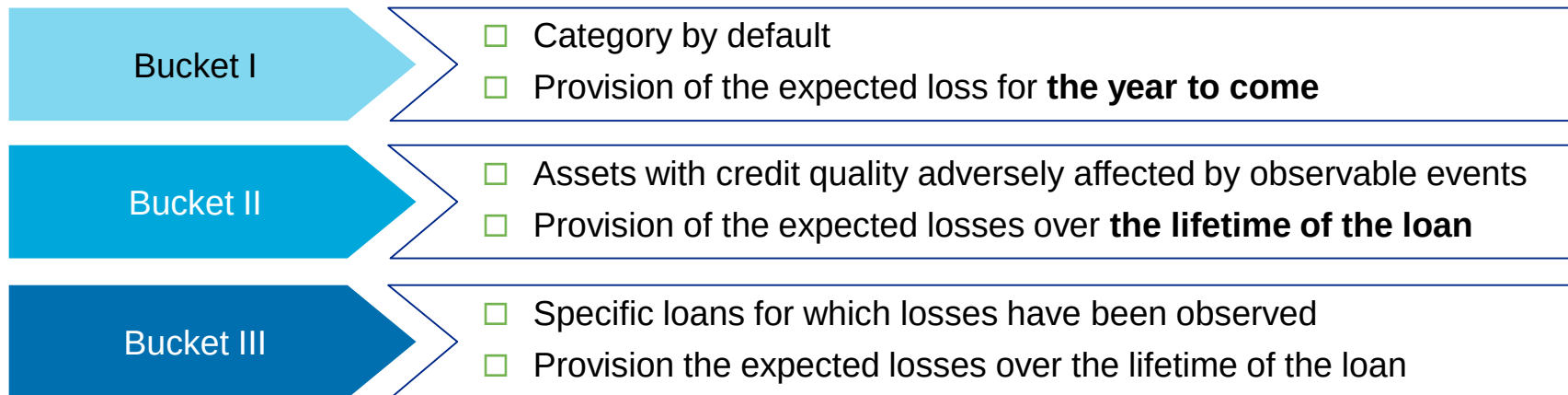
To predict the capital ratio the models are used to predict the Risk Weighted Assets and capital impacts of credit risk

Models are combined with the regulatory and accounting rules

- In a simplified view, the **Risk Weighted Assets** ($RWA = RW \times EAD$) depend on the rating via the Risk Weight; in a more complex, advanced view, the Risk Weighted Assets are more risk sensitive with a more refined dependency on rating, PD, LGD, EAD.

Rating dependency	Classification	Credit Quality Step (CQS)							Obligor in default (D)
		AAA/AA 1	A 2	BBB 3	BB 4	B 5	CCC 6	NR NR	
Obligor rating	Rated corporates & project finance	20%	50%	100%	100%	150%	150%		100% RW if provision rate above 20%
Obligor rating	Rated RGLA & PSE	20%	50%	50%	100%	100%	150%		150% RW if provision rate
Obligor rating	Rated banks	20%	50%	50%	100%	100%	150%	100%	
Obligor rating	Rated banks with effective maturity less than one quarter	20%	20%	20%	50%	50%	150%	100%	

- For **provisions**, credits are classified in three buckets:



- For bucket 1 and 2, one needs to estimate 1-year and lifetime expected credit losses; with a **forward looking macro-economic view. For the actual bucket 3, the provision is determined by the LGD.**
- For predicting the evolution of the provisions and corresponding Profit & Loss impact, one predicts the migrations between buckets (a.o. on rating migrations) and the corresponding provision amount given the predicted bucket, rating and other risk parameters.

The application of the models for the multiperiod view combines the model outcomes with the regulatory and accounting logic

Methodology overview and application to stress testing given the scenario

Given the base and/or stress case macro-economic scenario, one is interested to know the KPI (RWA, losses, ..., provision level, liquidity reserves) for the next years.

Calculation methodology (analytical or Monte Carlo calculation)

$$\text{Loss}(t) = \sum_{r_{t-1}} \mathbb{P}[r_0 \rightarrow r_{t-1}] \cdot \text{LGD}(r_{t-1}) \cdot \mathbb{P}[r_{t-1} \rightarrow \text{default}] \cdot \text{EAD}_{t-1:t}$$

$$\text{RWA}(t) = \sum_{r_{t-1}} \mathbb{P}[r_0 \rightarrow r_t] \cdot \text{RW}(r_t) \cdot \text{EAD}_t$$

...

$$\text{Prov}(t) = \sum_{r_{t-1}} \mathbb{P}[r_0 \rightarrow r_t] \cdot \text{Prov}(r_t) \cdot \text{EAD}_t$$

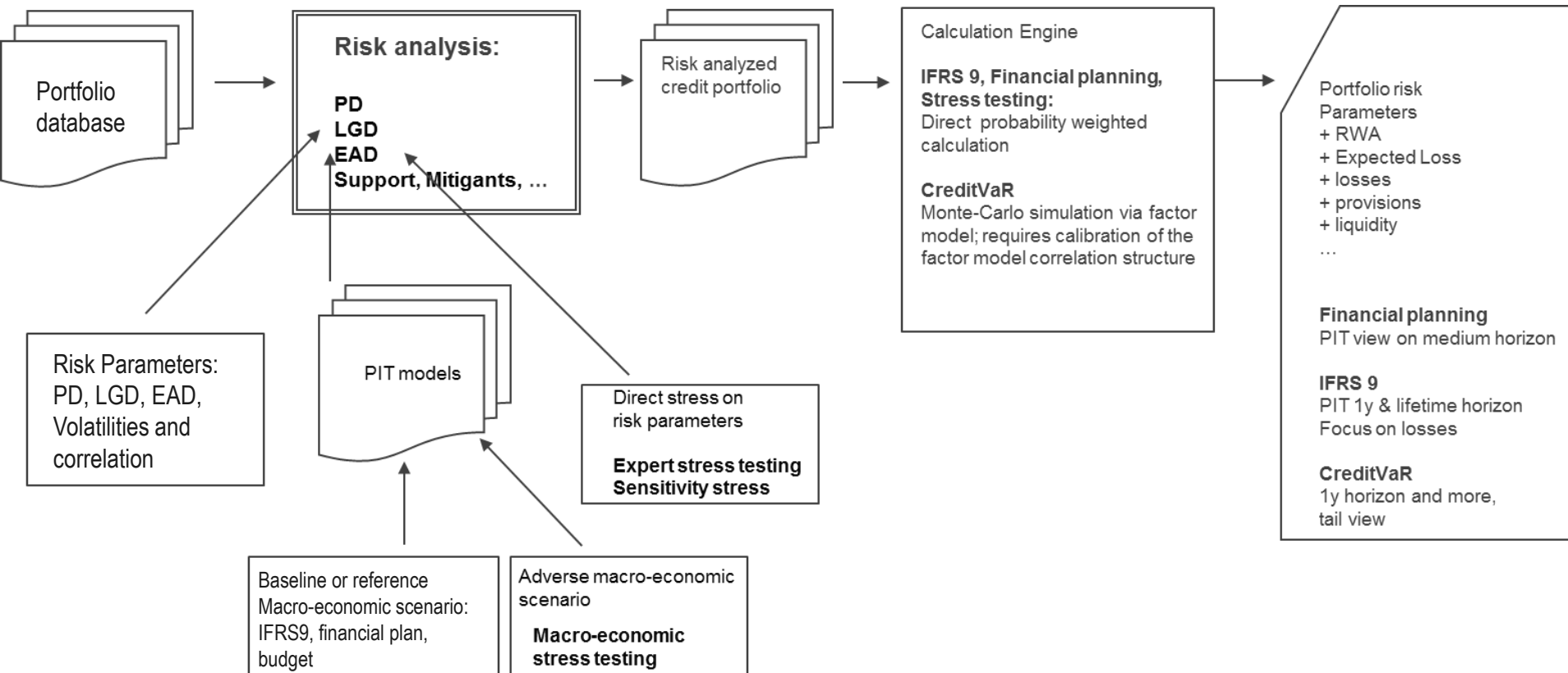
Specific elements for provisioning:

- The main drivers for the future IFRS9 provisions are the rating and the probability to be in bucket I, II or III.
- The probability to be in a rating r_t is obtained from the generator migration matrix
- The probability to be in bucket I, II and III also depends (partially) on the generator migration matrix

Type of ST	Sensitive parameters
Historical ST	Risk of choosing the wrong crisis. ...com crisis on a investment grade? Banking Financial crisis in a non investment portfolio? Worst GDP of last years
Expert Based ST	Model updates & recalibration Downgrades of 1, 2 or more notches on which counterparts Increase in LGD less intuitive than downgrades
Credit VaR models	Latent variable model aiming at sensitive tail risk modelling Difficult to estimate the correlations
<u>Macroeconomic Portfolio Models</u>	Macroeconomic scenario/forecast Correlation of macro-economy with risk parameters

The application of the models for the multiperiod view combines the model outcomes with the regulatory and accounting logic

Stress testing, financial planning, budget, pillar 2 ICAAP, ...



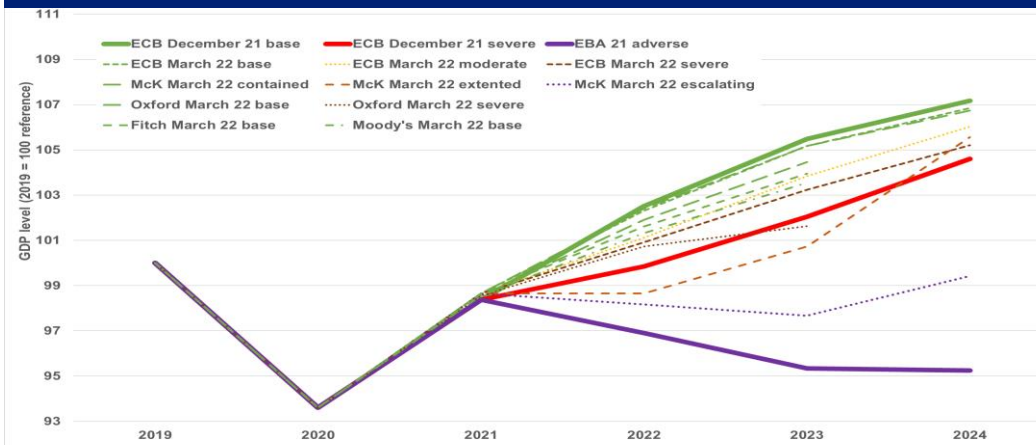
In the integrated calculation process serves multiple purposes such as stress testing and the financial plan. To each line or homogeneous pool in the credit portfolio database, one assigns the appropriate risk parameters. On the completed database, the concerned KPI are calculated. All these models and parameters require proper governance.

The evaluation of stress scenarios serves to identify potential vulnerabilities and the corresponding action plans

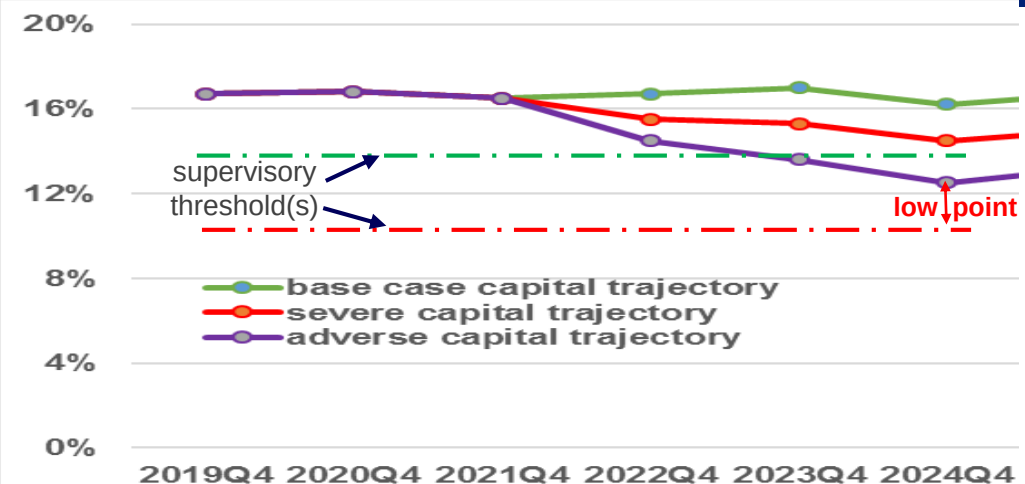
Scenario → Model → Prediction → Conclusions

- One starts with the **definition of stress scenarios** that are relevant in the actual context, a.o., Russian invasion, inflation and interest rate increase.
- The evaluation of the scenario is made in a global view, taking into account the **credit** impact, but also the **market** impacts (foreign exchange rates, long term rates, inflation, credit spreads) and **operational risk**
- The impact of each scenario is evaluated on regulatory and economic capital required. The main **root causes** of the impacts are evaluated and, when required, **remedial actions** are **considered** to improve the margin (low point) with respect to the applicable supervisory threshold within the **risk appetite** levels.

EU GDP projections



Capital ratio projections (example)



Conclusion, Questions & Answers

Models are challenging to develop, apply and use for financial decision making

Thank you!