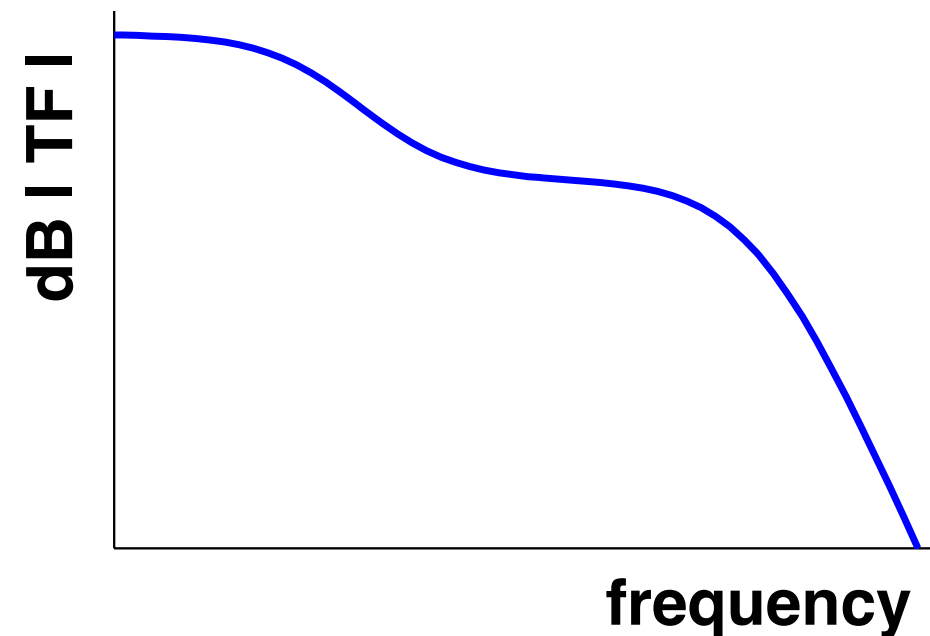
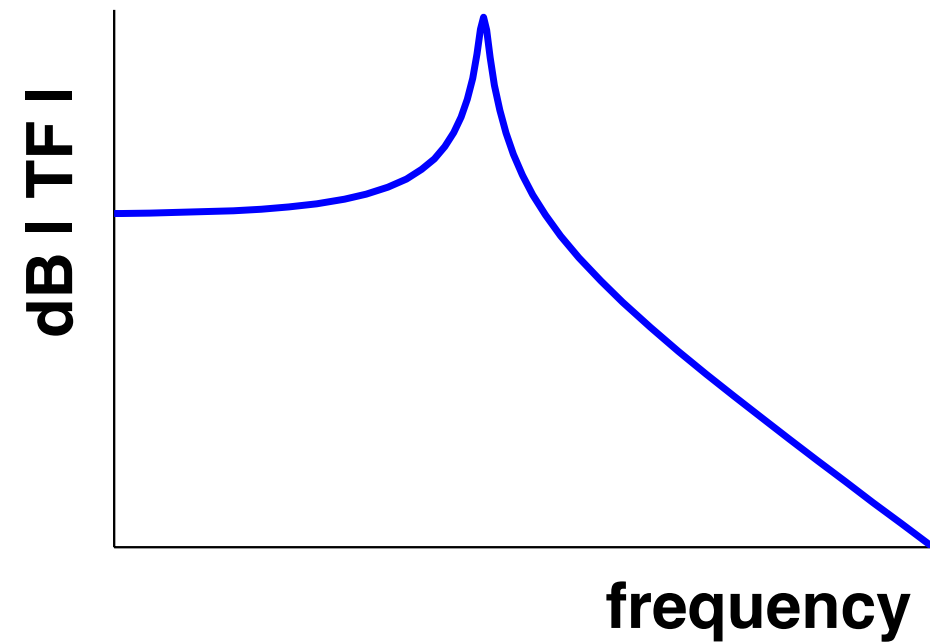
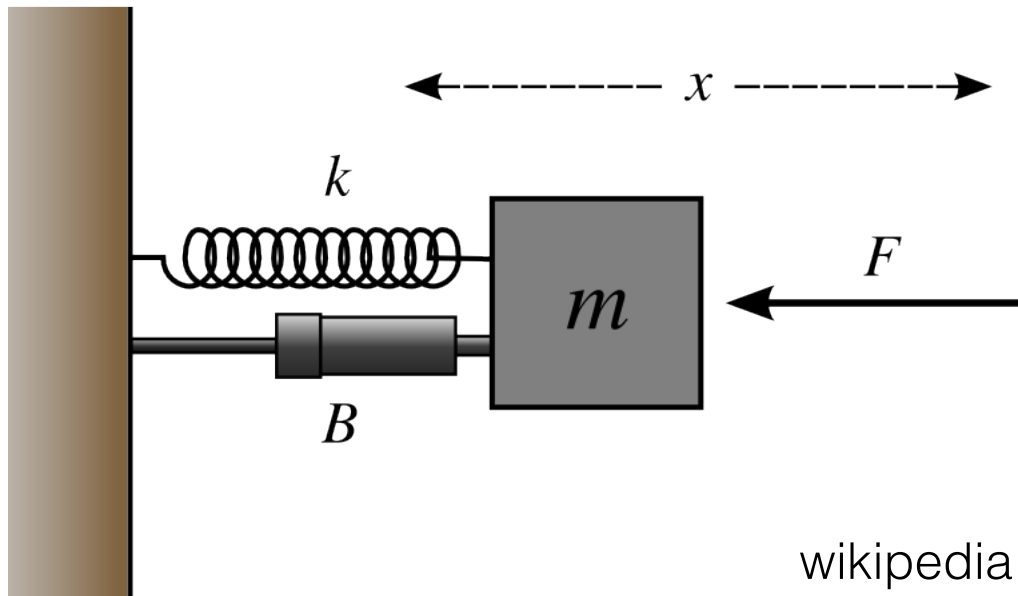


Kernel-based Transfer Function Estimation with Enhanced Prior Knowledge

John Lataire

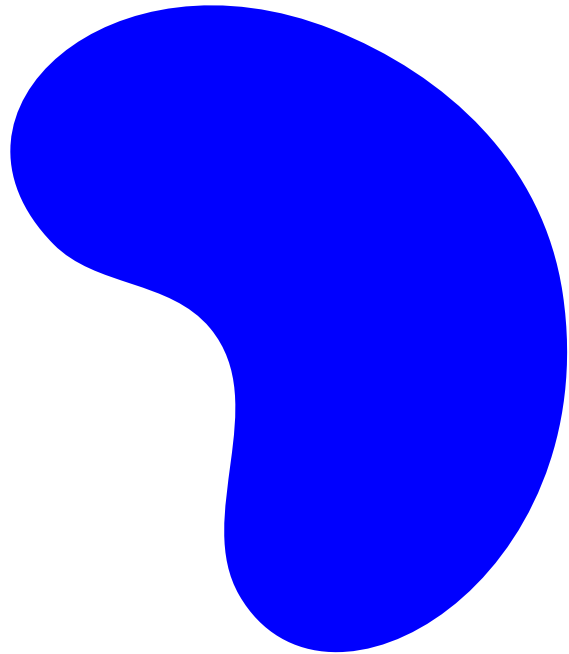
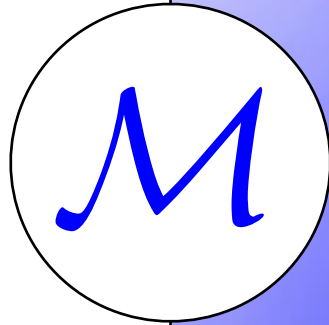


Transfer Functions



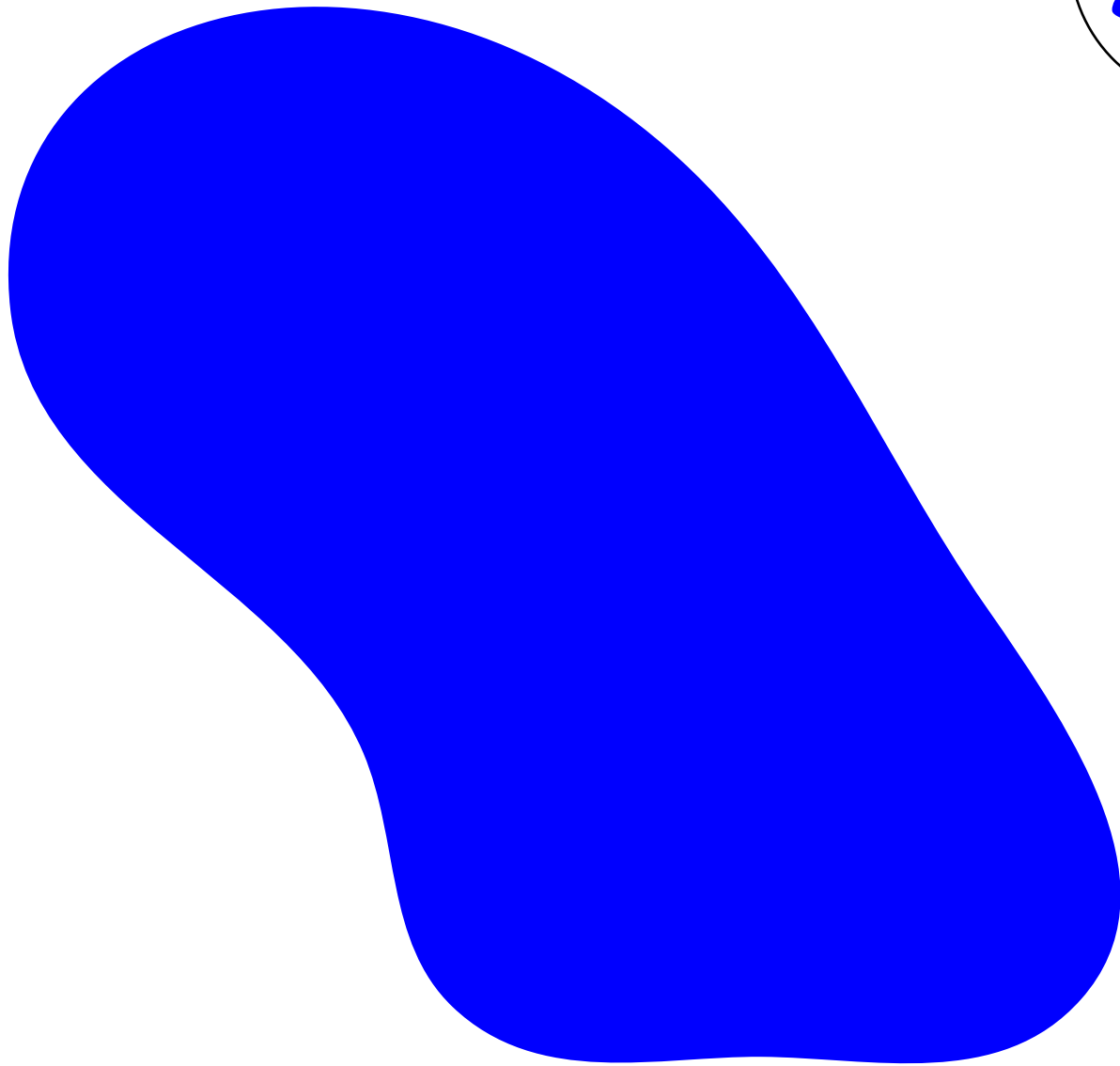
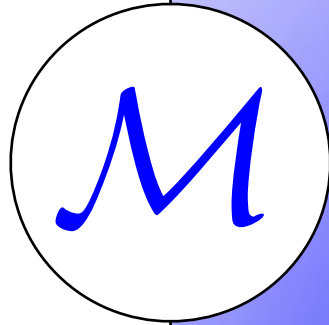
Classical regression

Kernel methods



Classical regression

Kernel methods

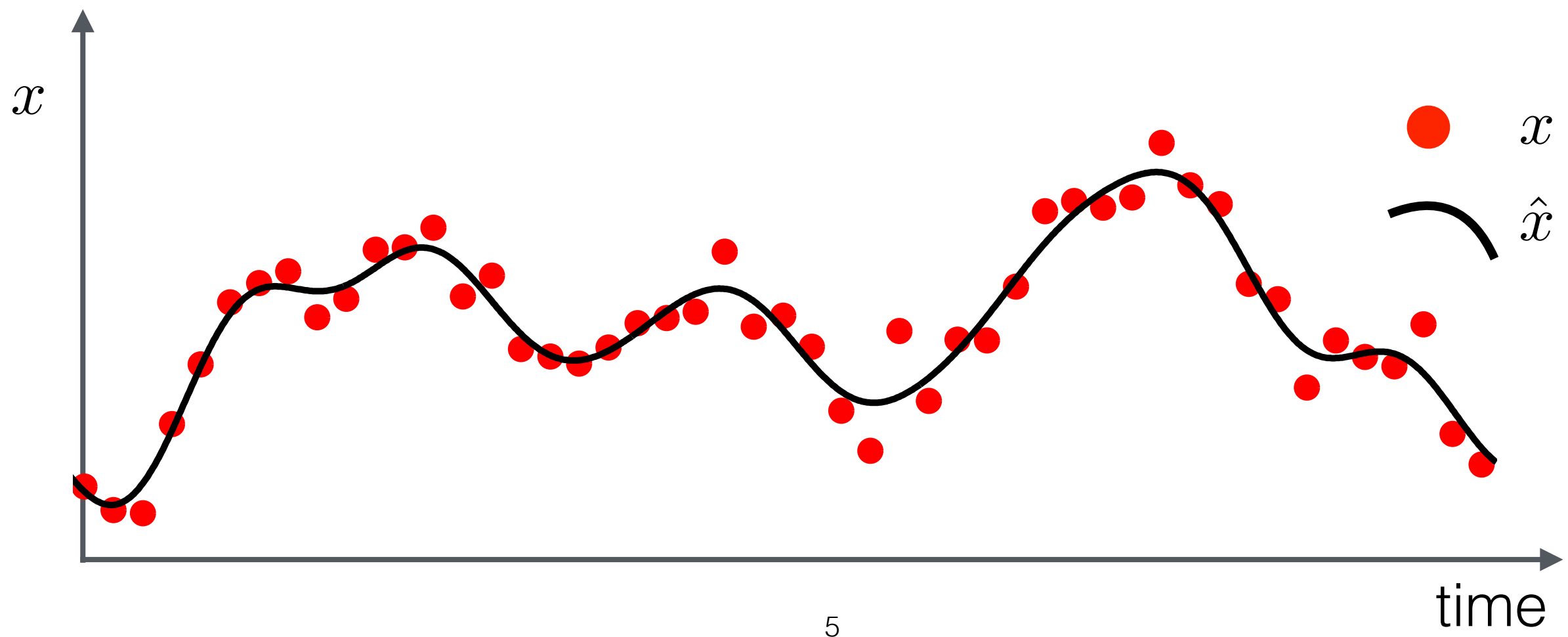


Linear Least Squares

$$x = x_o + e$$

ρ , such that $\|x - \hat{x}\|_2^2$ is minimized

$$\hat{x} = \phi\rho$$



Linear Least Squares

$$x = x_o + e$$

ρ , such that $\|x - \hat{x}\|_2^2$ is minimized

$$\hat{x} = \phi \rho$$

Solution

$$\rho = (\phi^\top \phi)^{-1} \phi^\top x$$

$$\hat{x} = \phi (\phi^\top \phi)^{-1} \phi^\top x$$

$$\hat{x} = \phi \rho$$

Linear Least Squares + regularization

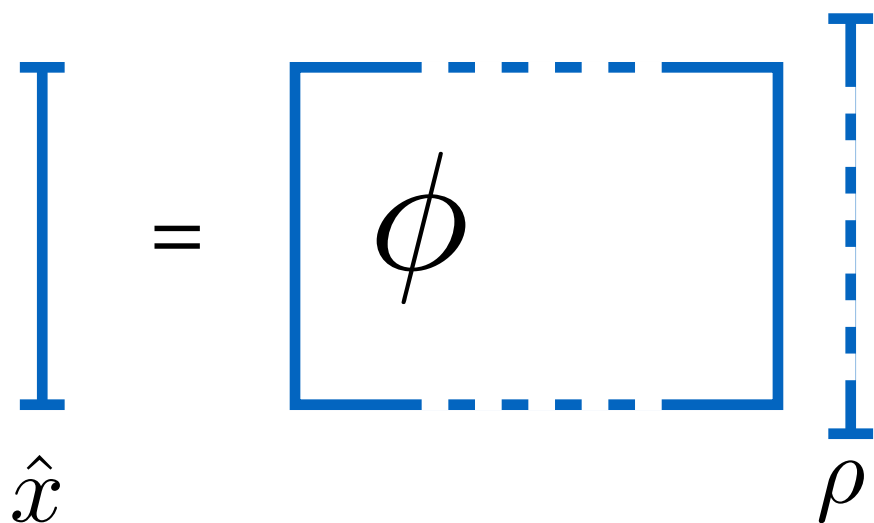
$$x = x_o + e \quad \rho, \text{ such that } \|x - \hat{x}\|_2^2 + \frac{\rho^\top \rho}{\gamma} \text{ is minimized}$$

$$\hat{x} = \phi \rho$$

Solution

$$\hat{x} = \underbrace{\phi \phi^\top (\phi \phi^\top + I \gamma^{-1})^{-1}}_{\alpha} x$$

∞ basis functions



Linear Least Squares + regularization

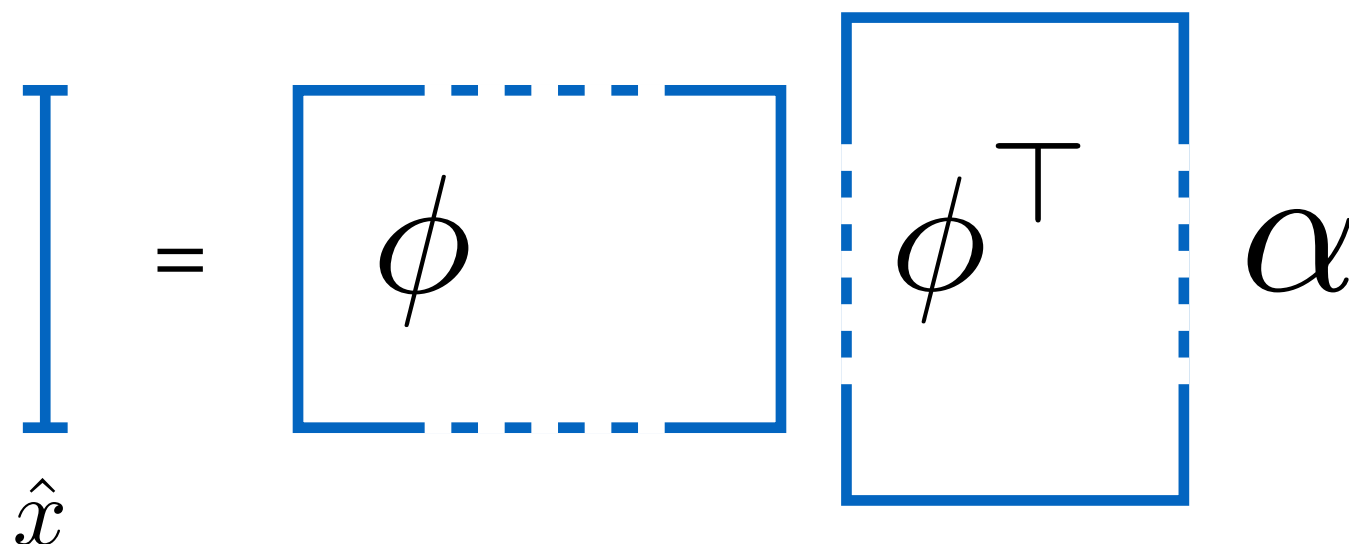
$$x = x_o + e \quad \text{such that} \quad \|x - \hat{x}\|_2^2 + \frac{\rho^\top \rho}{\gamma} \quad \text{is minimized}$$

$$\hat{x} = \phi \rho$$

Solution

$$\hat{x} = \underbrace{\phi \phi^\top (\phi \phi^\top + I \gamma^{-1})^{-1}}_{\alpha} x$$

∞ basis functions



Linear Least Squares + regularization

$$x = x_o + e \quad \text{such that} \quad \|x - \hat{x}\|_2^2 + \frac{\rho^\top \rho}{\gamma} \quad \text{is minimized}$$

$$\hat{x} = \phi \rho$$

Solution

$$\hat{x} = \phi \phi^\top \underbrace{(\phi \phi^\top + I \gamma^{-1})^{-1}}_{\alpha} x$$

∞ basis functions

$$\begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} = \boxed{K} \alpha$$

$\hat{x}$

Linear Least Squares + regularization

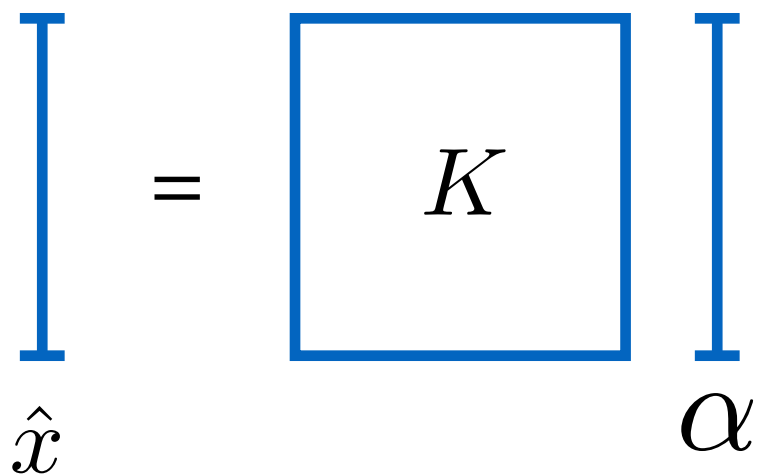
$$x = x_o + e \quad \text{such that} \quad \|x - \hat{x}\|_2^2 + \frac{\rho^\top \rho}{\gamma} \quad \text{is minimized}$$

$$\hat{x} = \phi \rho$$

Solution

$$\begin{aligned} \hat{x} &= \phi \phi^\top (\phi \phi^\top + I \gamma^{-1})^{-1} x \\ &= K (K + I \gamma^{-1})^{-1} x \end{aligned}$$

∞ basis functions

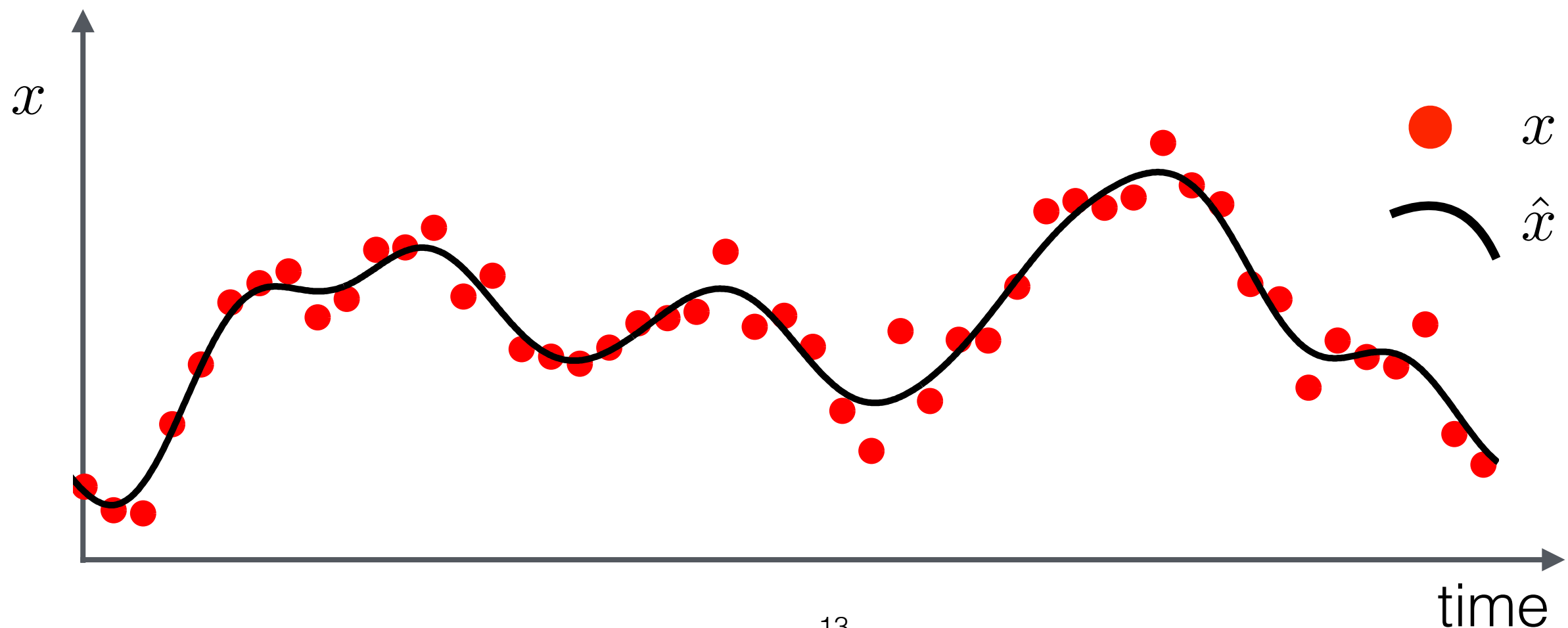

$$\hat{x} = K \alpha$$

Linear Least Squares + regularization

$x = x_o + e$ such that $\|x - \hat{x}\|_2^2 + \frac{\rho^\top \rho}{\gamma}$ is minimized

$$\hat{x} = \phi \rho$$

Solution $\hat{x} = K (K + I\gamma^{-1})^{-1} x$



Linear Least Squares + regularization

$x = x_o + e$ such that $\|x - \hat{x}\|_2^2 + \frac{\rho^\top \rho}{\gamma}$ is minimized

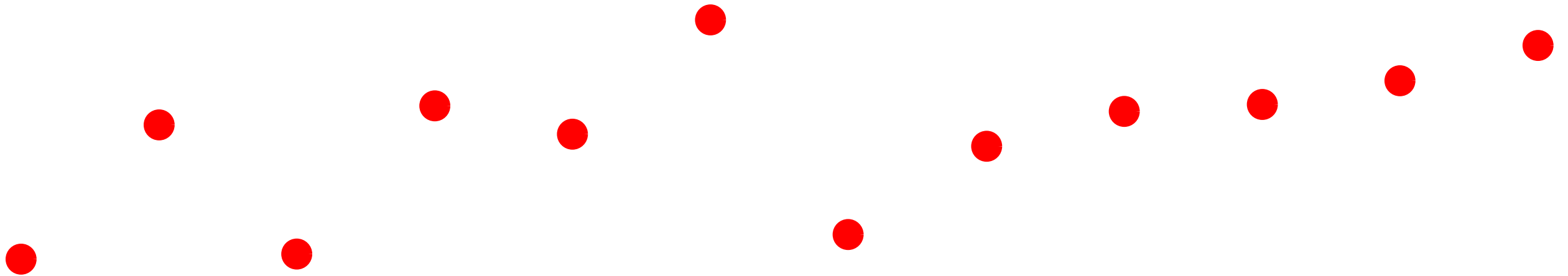
$$\hat{x} = \phi \rho$$

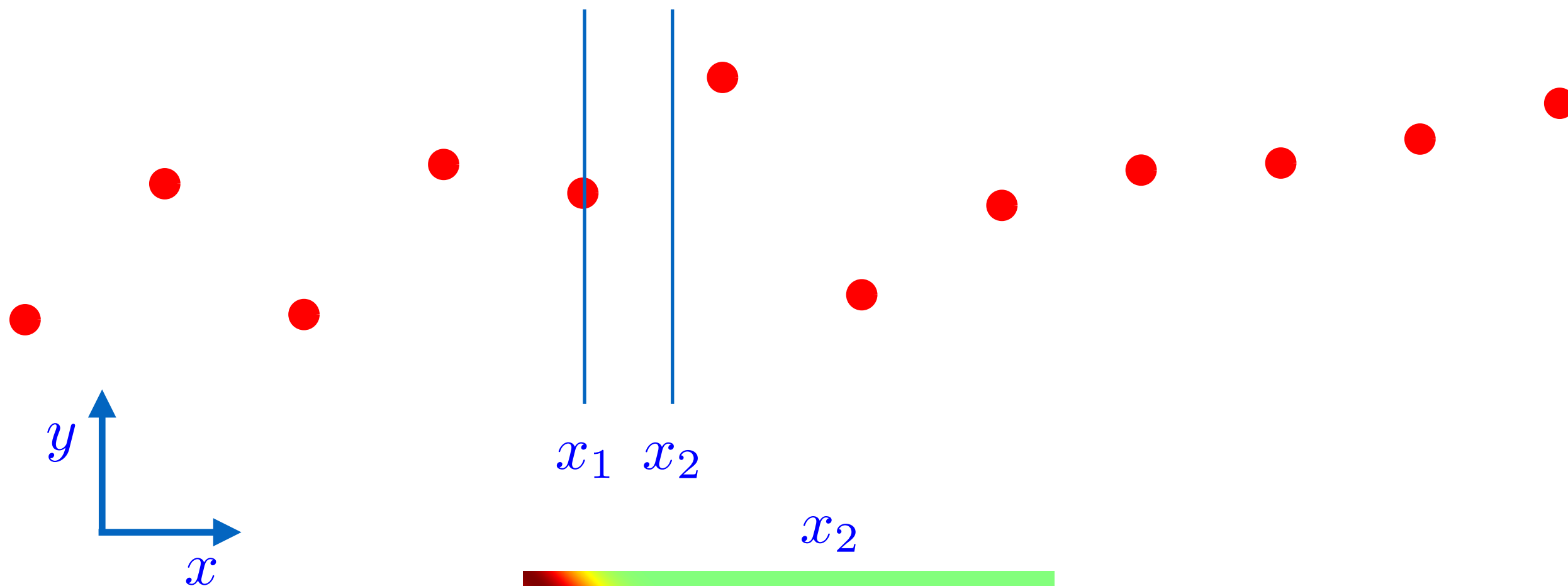
Solution $\hat{x} = K (K + I\gamma^{-1})^{-1} x$

Mercer's theorem (simplified)

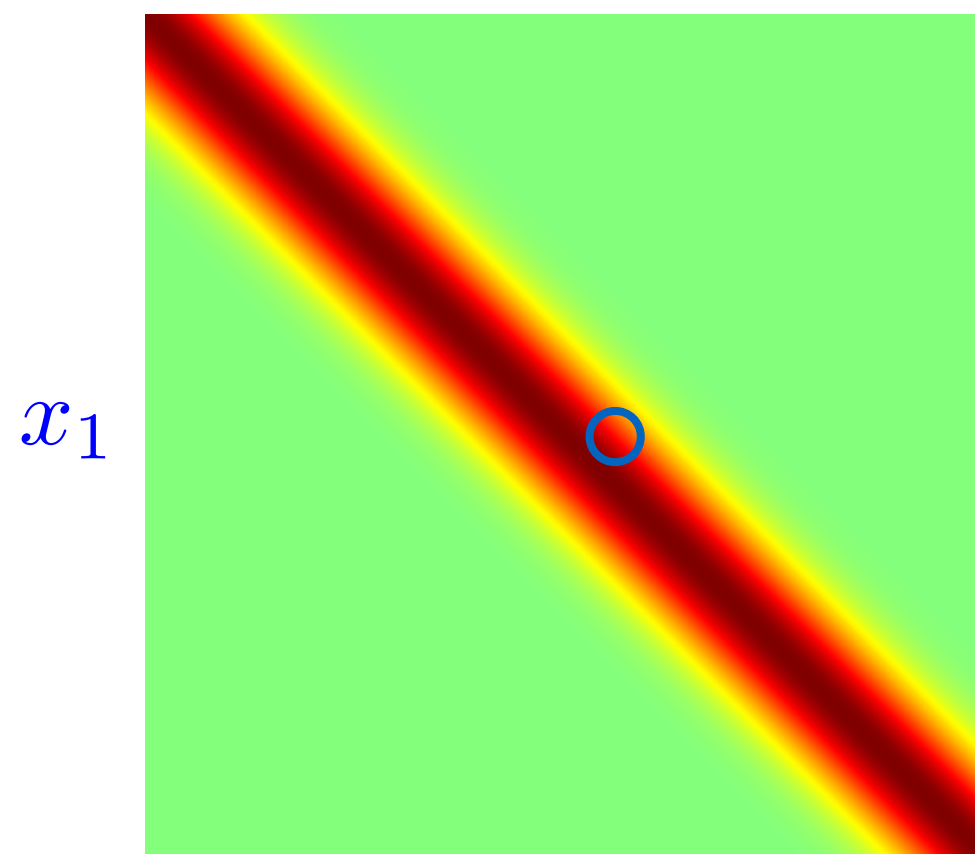
$$K = \phi \phi^\top \quad \Leftrightarrow \quad \begin{array}{ll} K \succeq 0 & \text{positive semi-definite} \\ K = K^\top & \text{symmetric} \end{array}$$

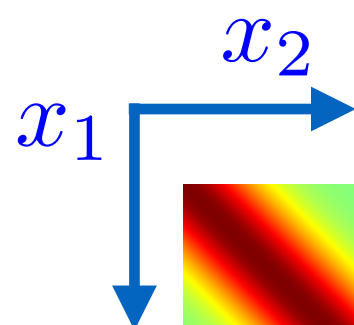
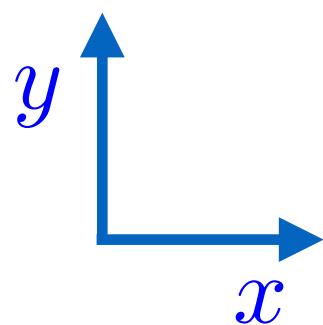
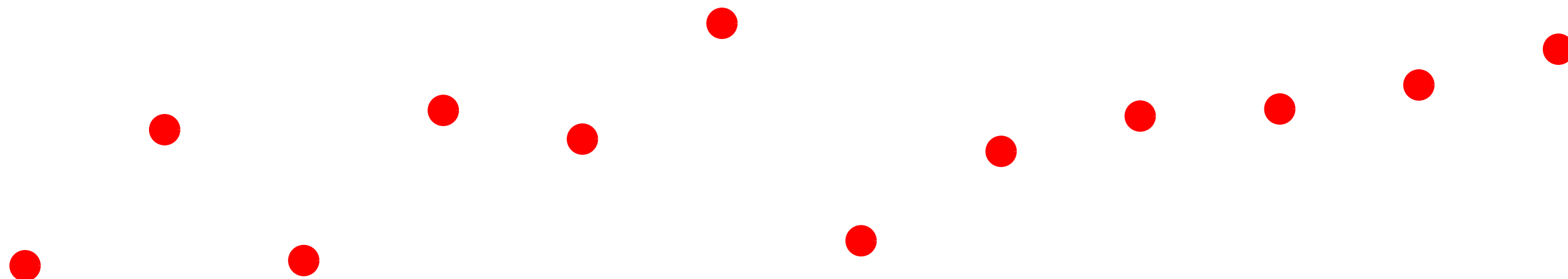
Gaussian Process





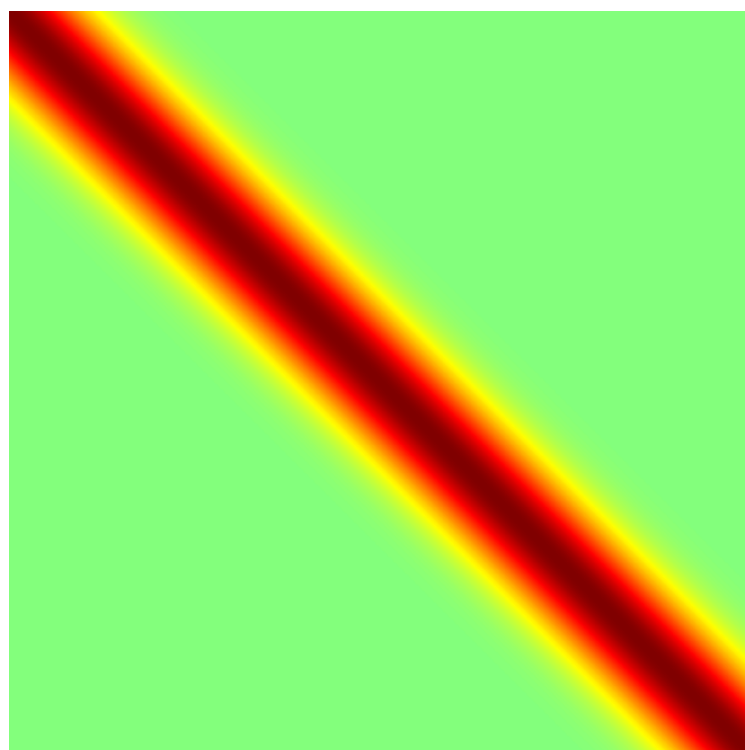
Covariance
functions

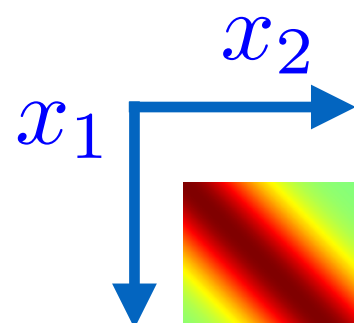
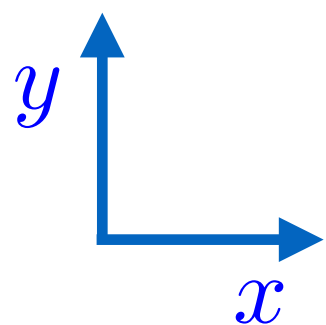
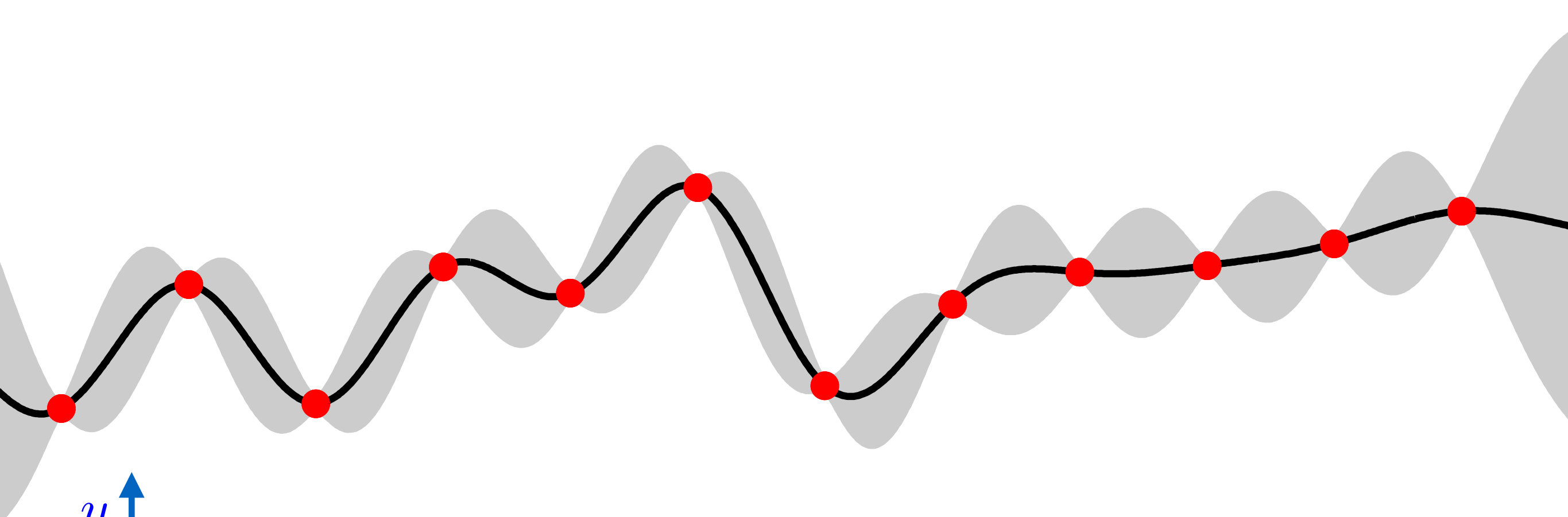




Covariance
functions

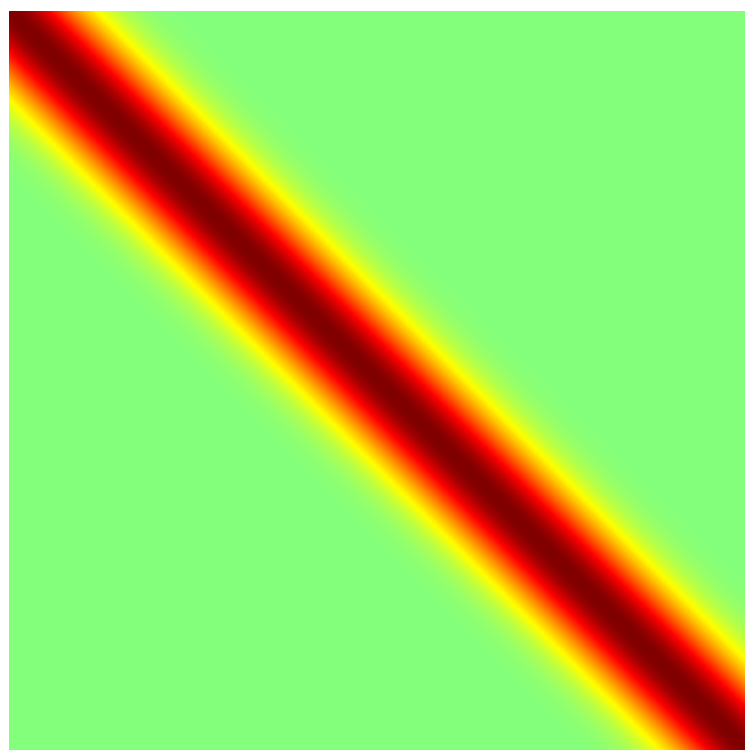
Prior



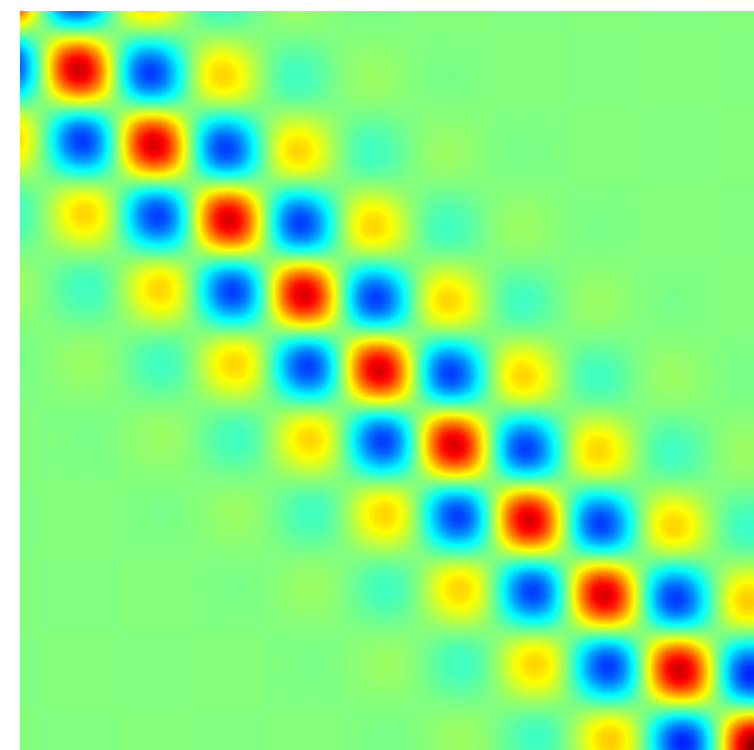


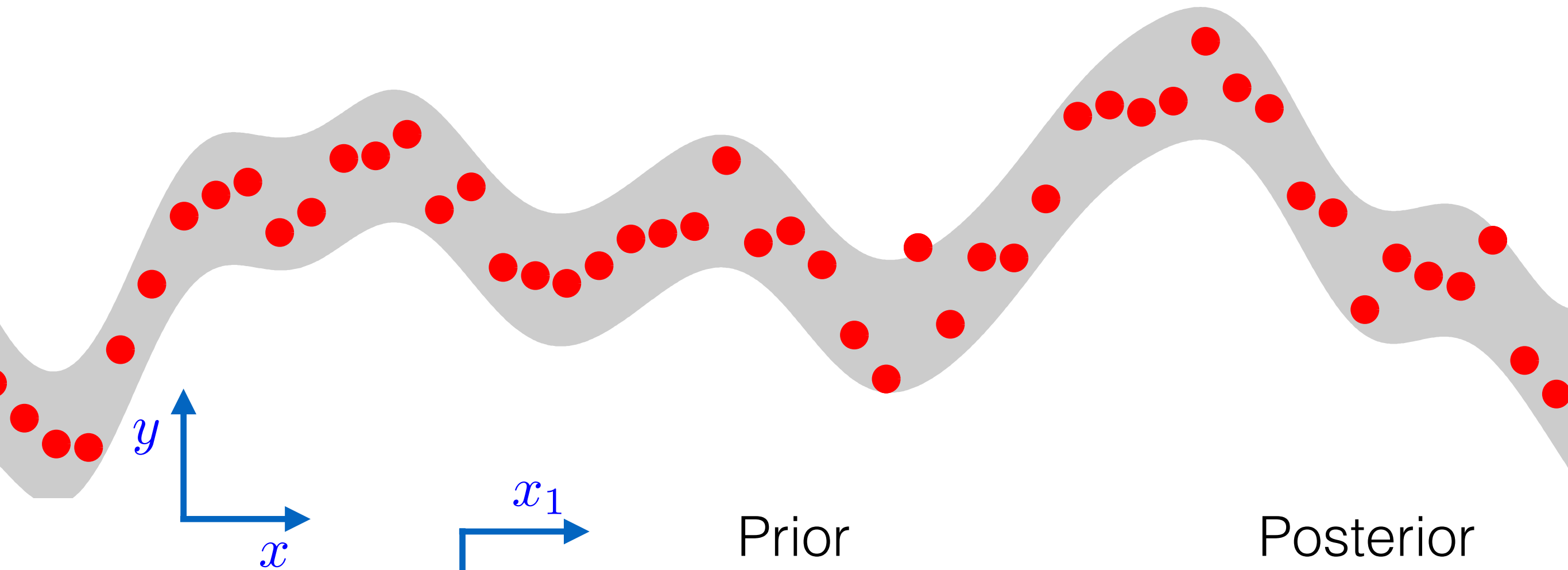
Covariance
functions

Prior



Posterior

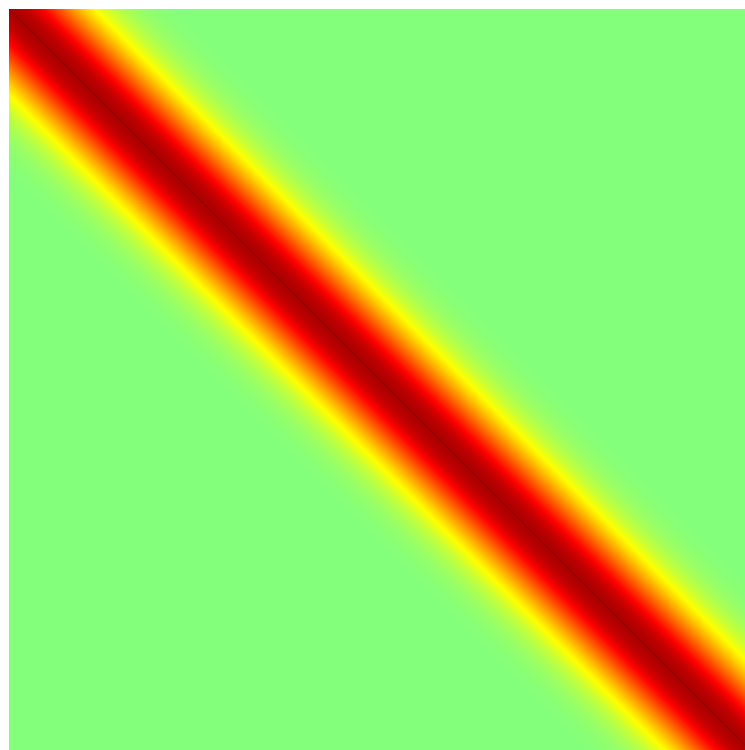




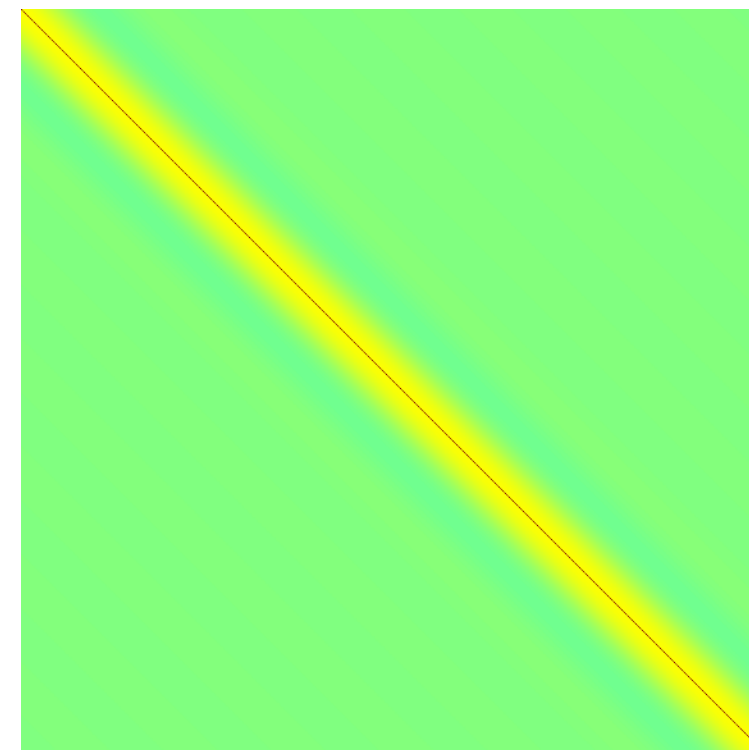
Covariance
functions

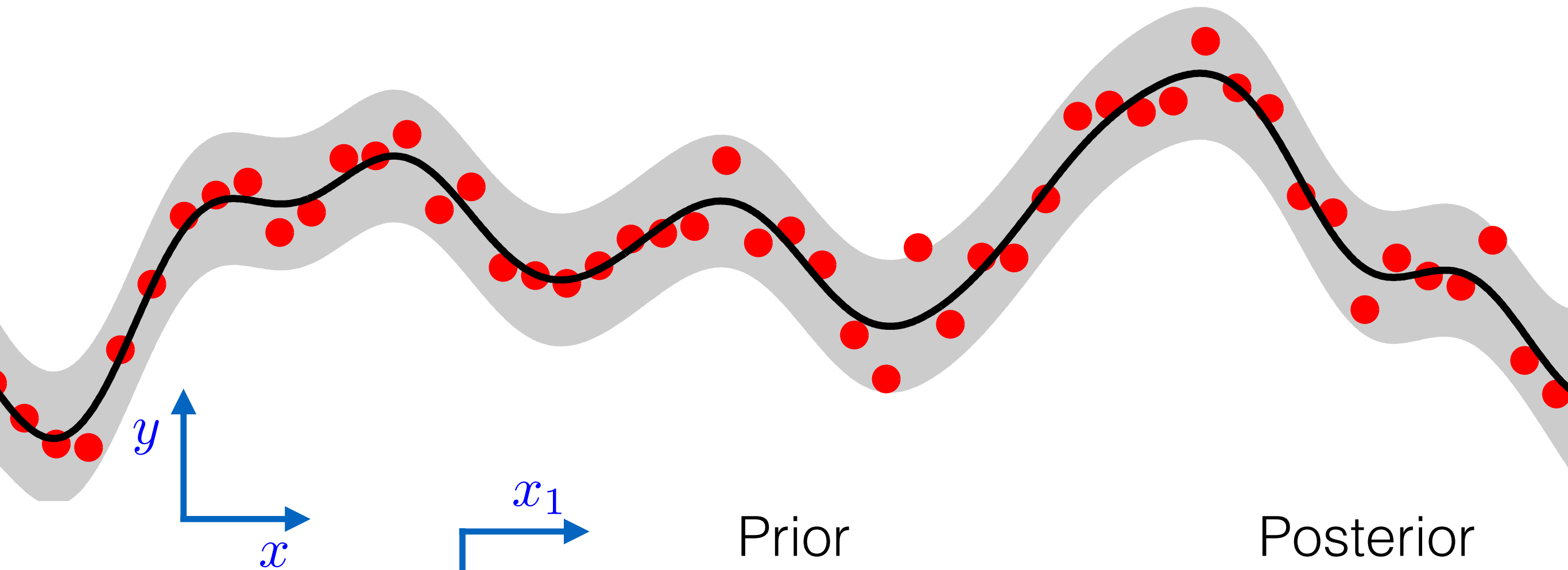
x_1
 x_2

Prior



Posterior

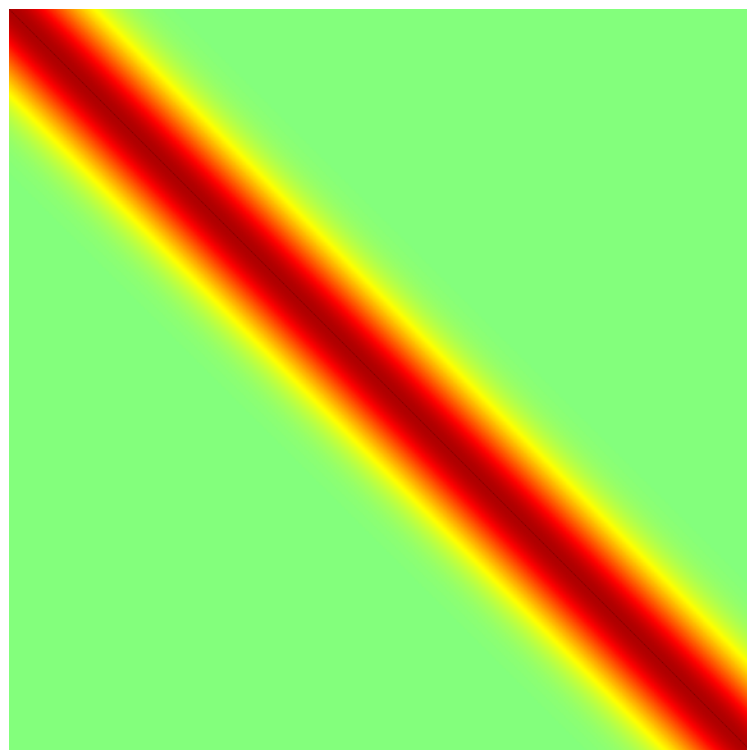




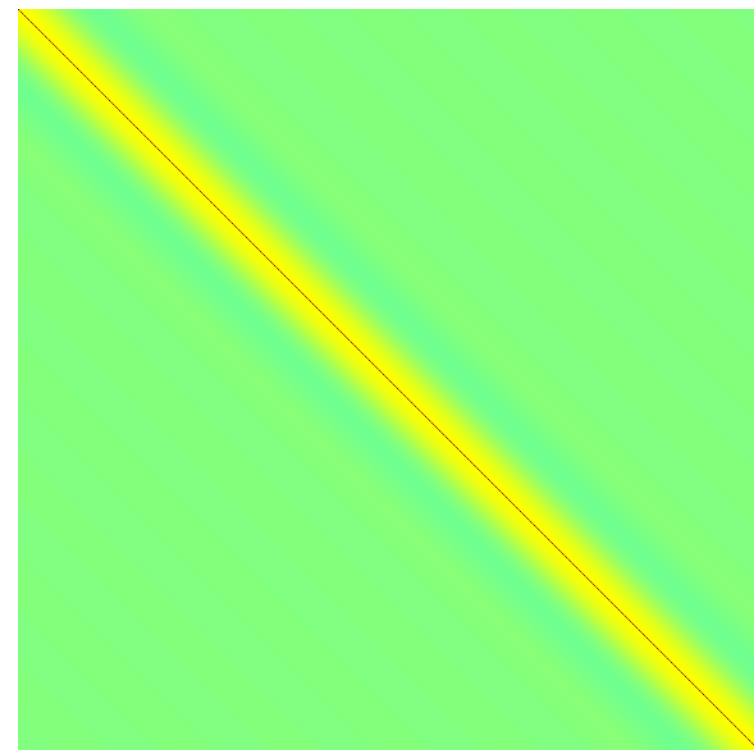
Covariance
functions

x_1
 x_2

Prior

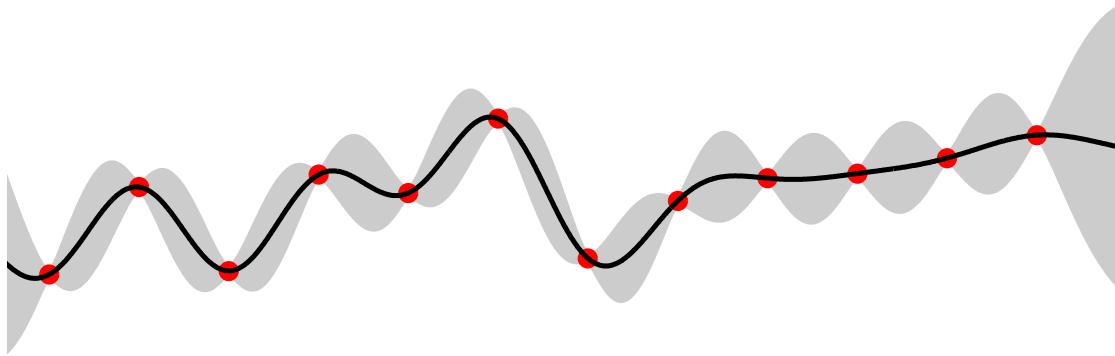


Posterior

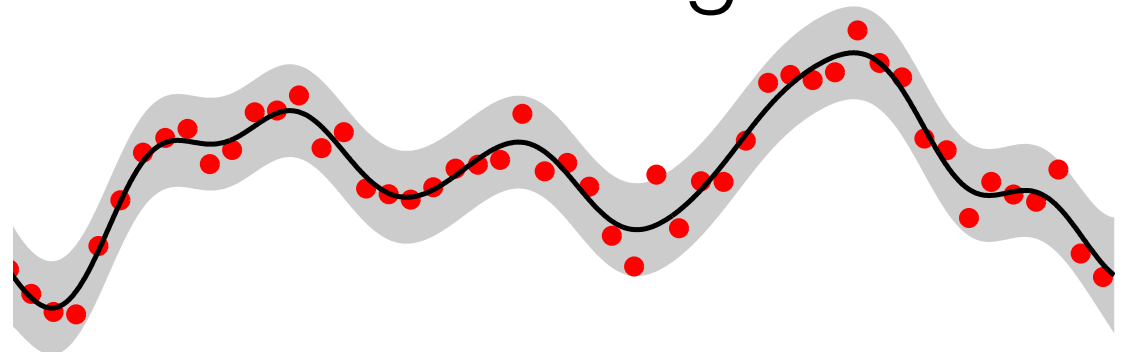


Gaussian Processes

Interpolation



Smoothing



Example Kernel

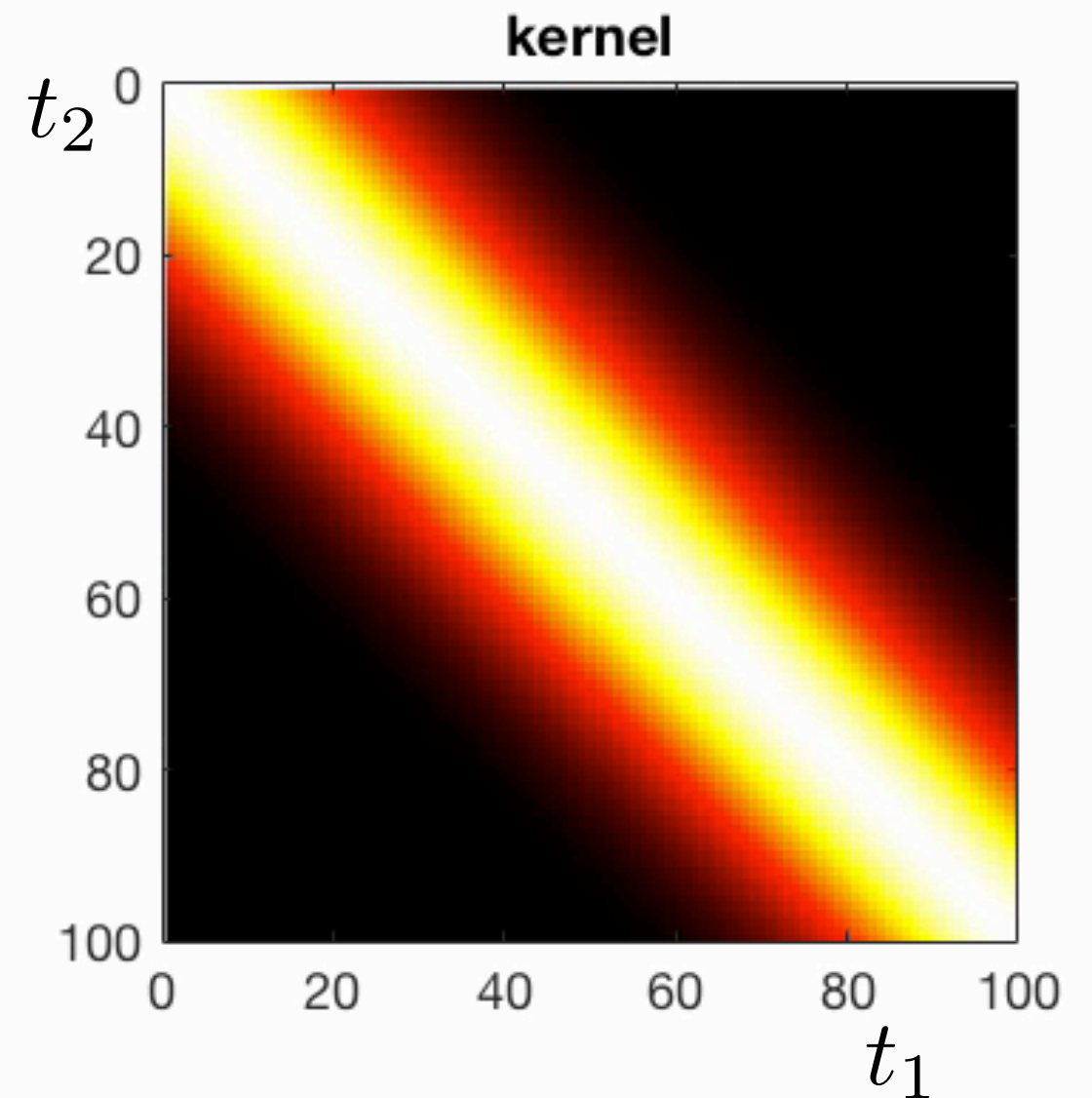
$$K = \phi\phi^\top$$

$$K(t_1, t_2) = e^{\frac{-(t_1 - t_2)^2}{l^2}}$$

Example Kernel

$$K = \phi\phi^\top$$

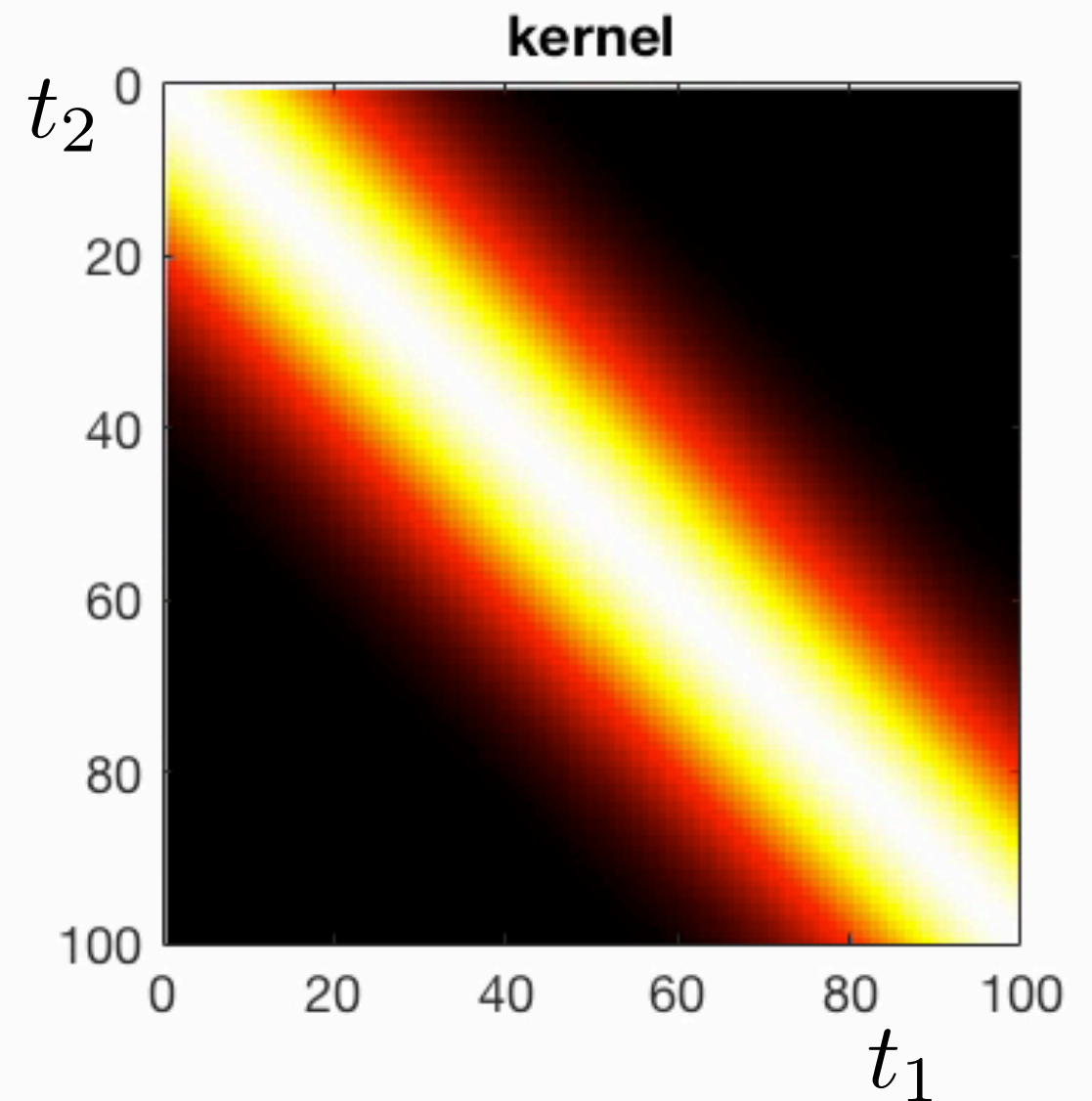
$$K(t_1, t_2) = e^{\frac{-(t_1 - t_2)^2}{l^2}}$$



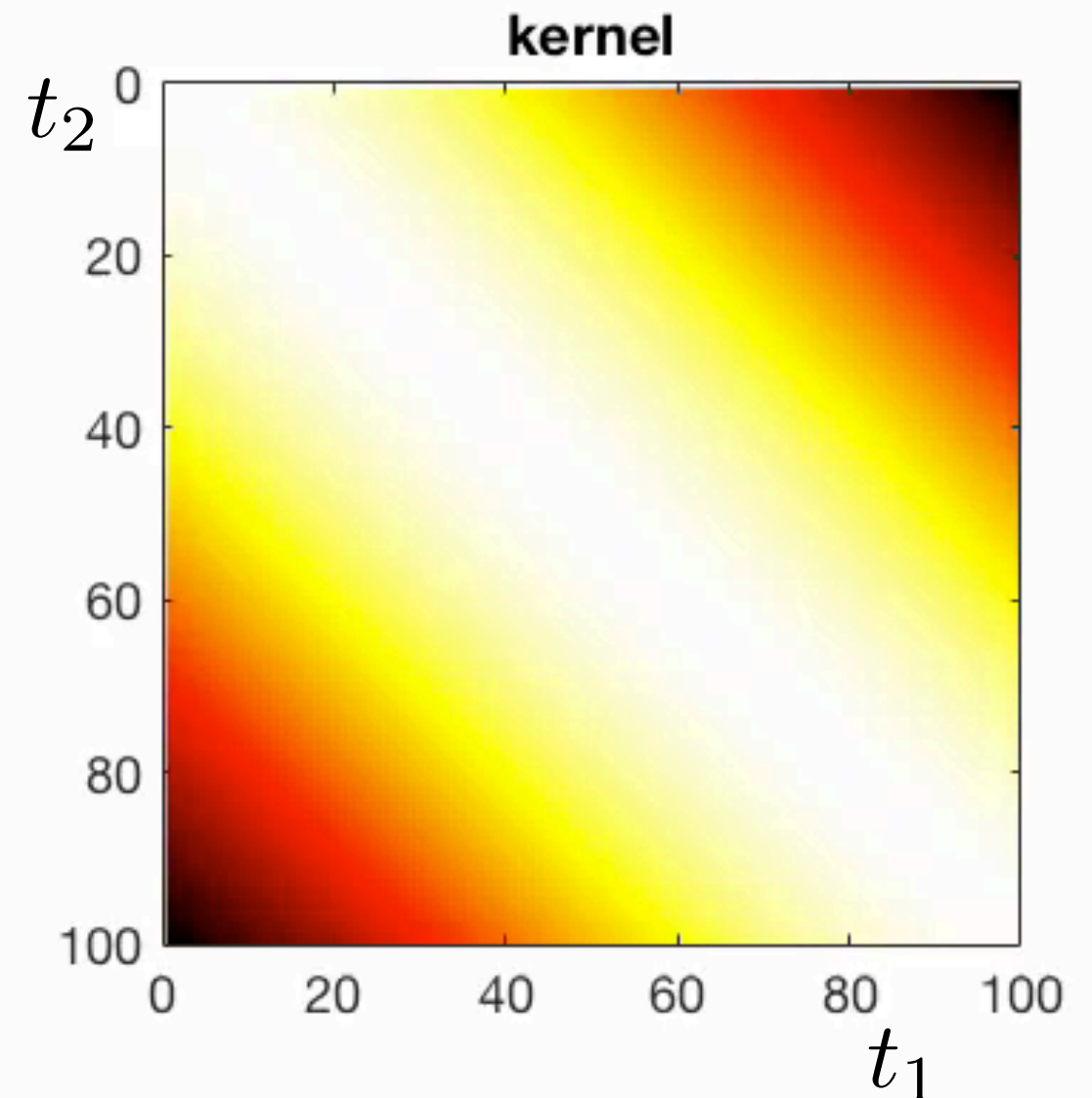
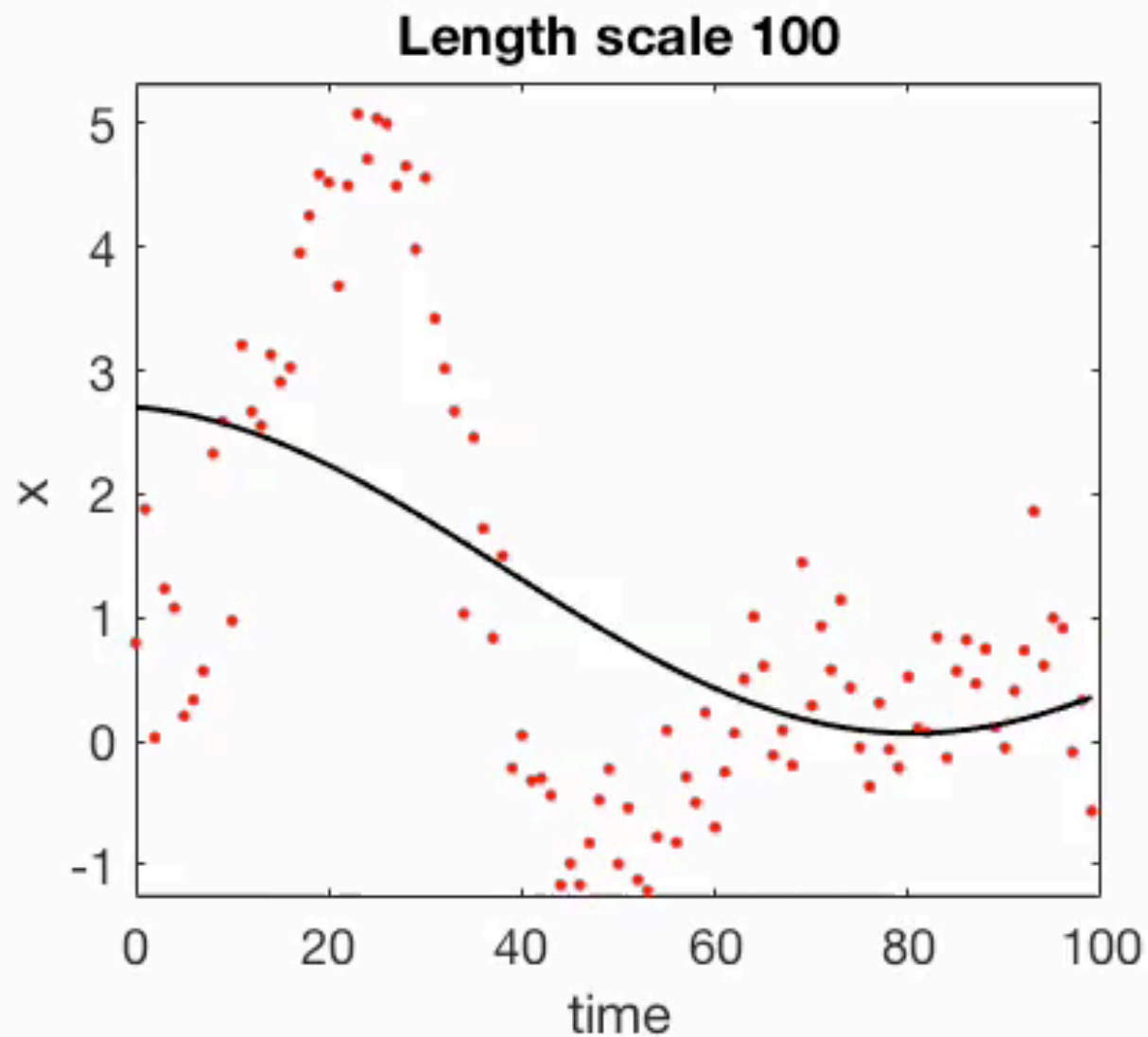
Continuous complexity tuning

$$K = \phi\phi^\top$$

$$K(t_1, t_2) = e^{\frac{-(t_1 - t_2)^2}{l^2}}$$



Continuous complexity tuning



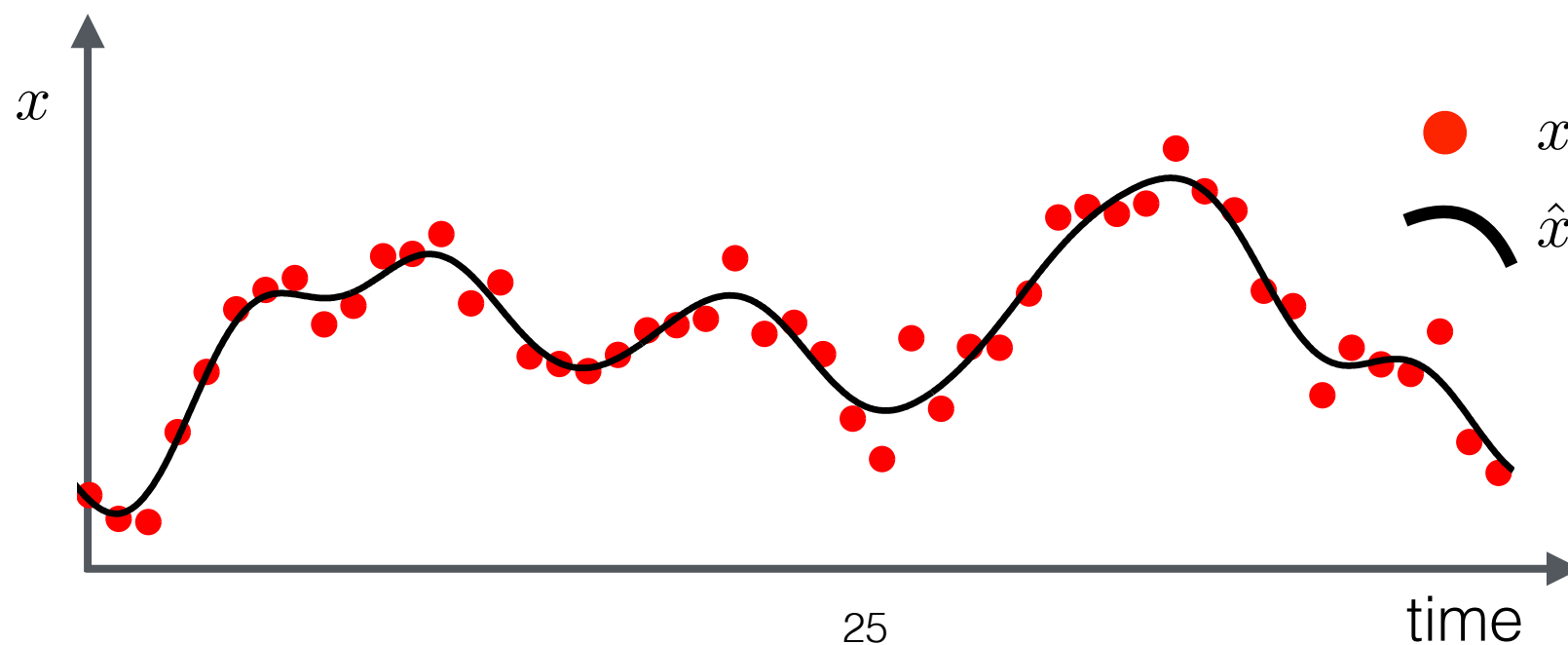
$$K(t_1, t_2) = e^{\frac{-(t_1 - t_2)^2}{l^2}}$$

Linear Least Squares

$$\hat{x} = \phi (\phi^\top \phi)^{-1} \phi^\top x$$

Kernel Based Regression

$$\hat{x} = K(K + \gamma^{-1}I)^{-1}x$$



Linear Least Squares

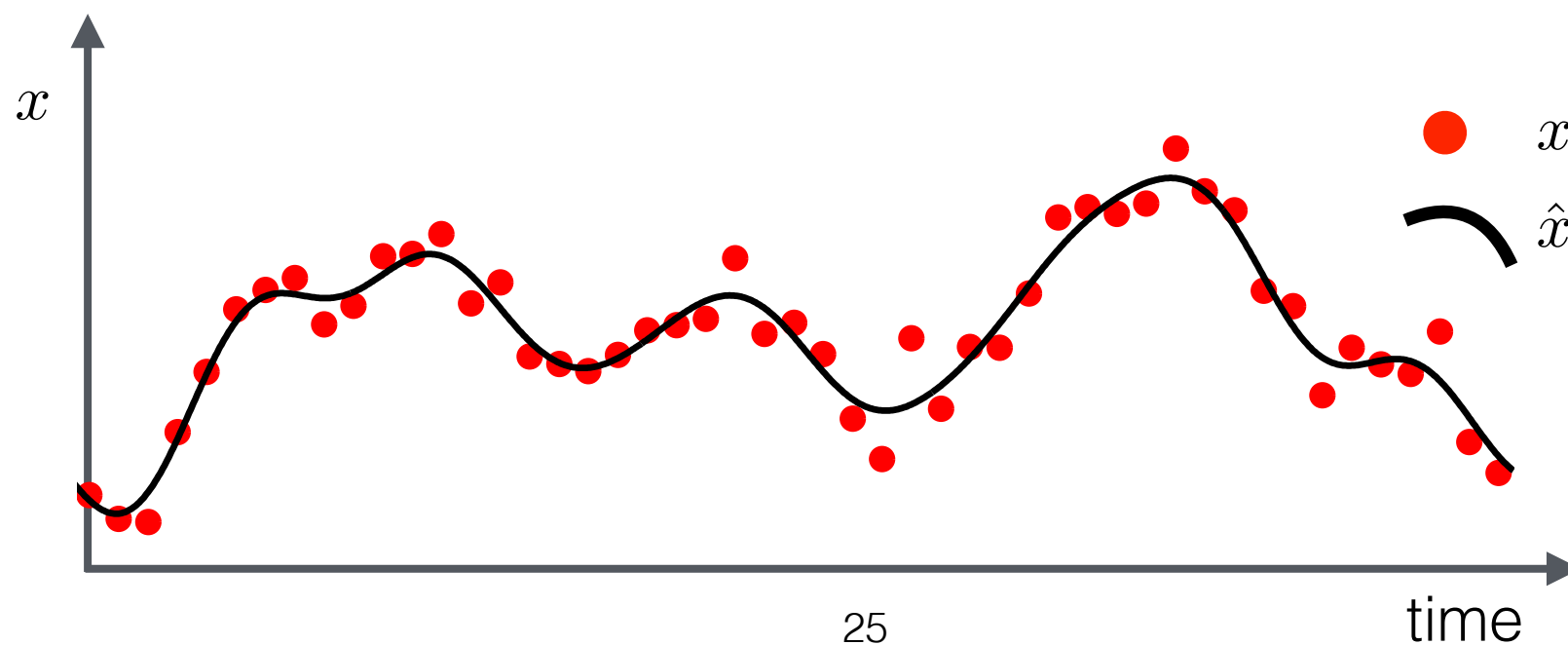
$$\hat{x} = \phi (\phi^\top \phi)^{-1} \phi^\top x$$

$$= U \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 0 & \\ & & & & 0 \end{pmatrix} U^\top x$$

Kernel Based Regression

$$\hat{x} = K(K + \gamma^{-1}I)^{-1}x$$

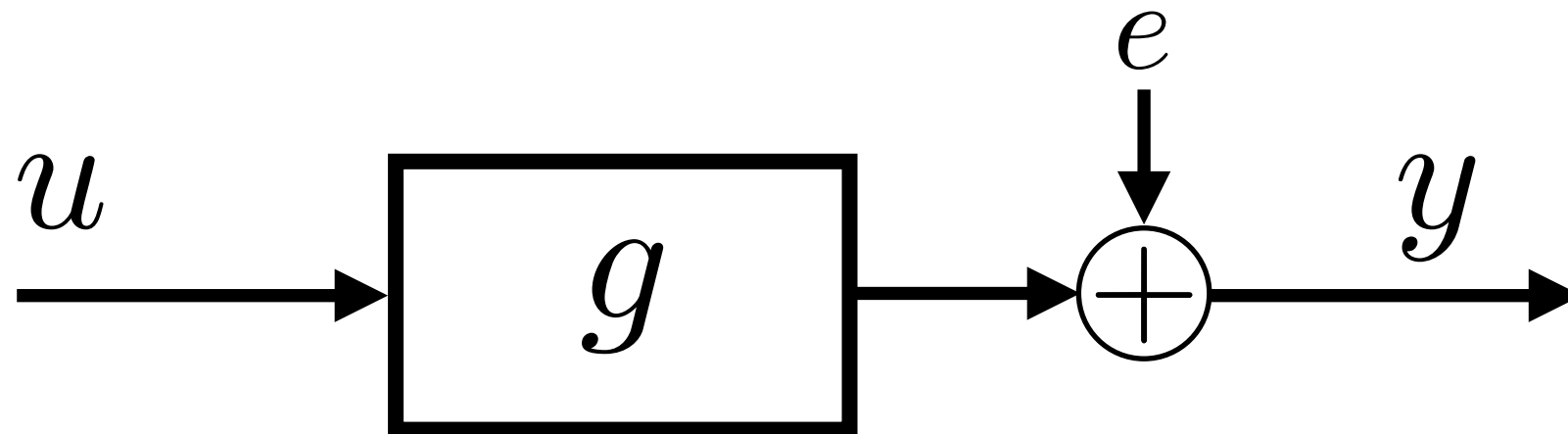
$$= U \begin{pmatrix} \ddots & & & \\ & \frac{s_i}{s_i + \gamma^{-1}} & & \\ & & \ddots & \end{pmatrix} U^\top x$$



Transfer Function Estimation

with Kernel methods
in the frequency domain

LTI system



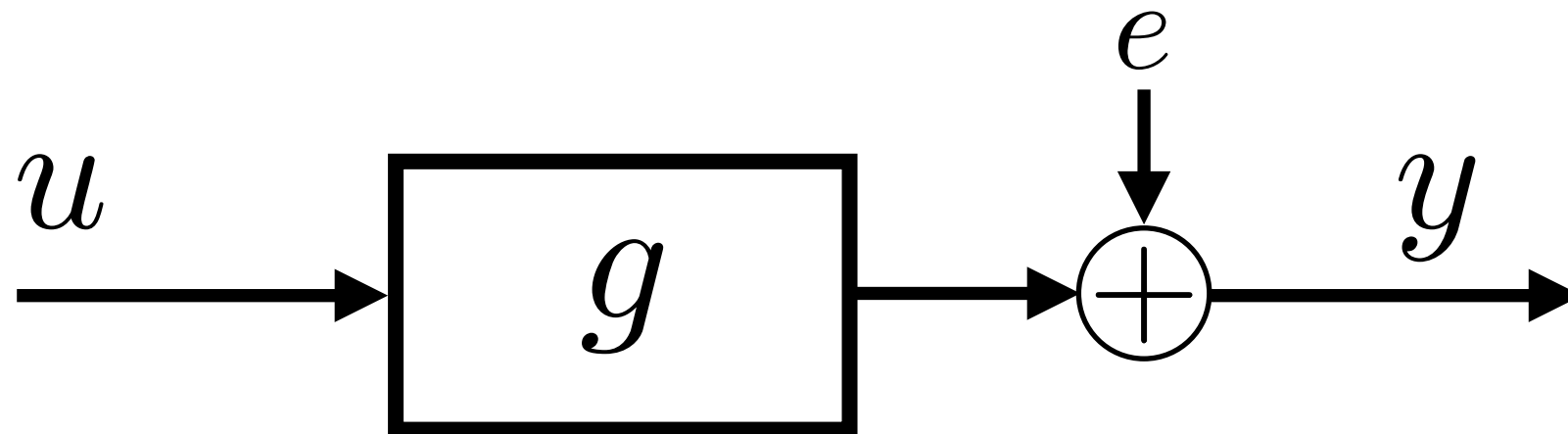
- Time domain

$$y(t) = (g * u)(t) + e(t)$$

- Frequency domain

$$Y(\Omega) = G(\Omega)U(\Omega) + E(\Omega)$$

LTI system



- Time domain

$$y(t) = (g * u)(t) + e(t) + \text{transient}$$

- Frequency domain

$$Y(\Omega) = G(\Omega)U(\Omega) + E(\Omega) + \text{Transient}$$

LTI system

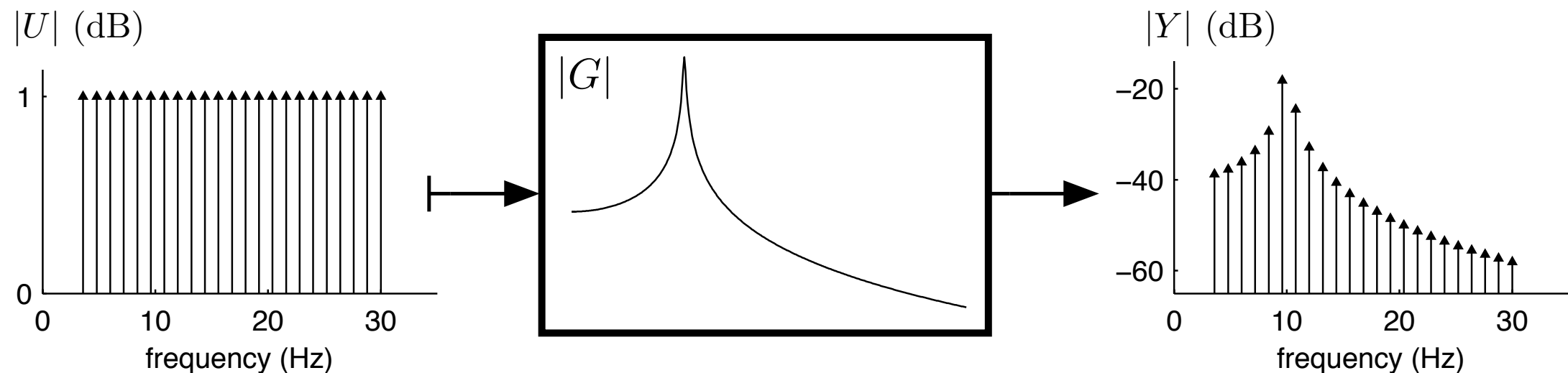
- Frequency domain **windowed, periodic signal**

$$Y(k) = G(\Omega_k)U(k) + E(k)$$

LTI system

- Frequency domain **windowed, periodic signal**

$$Y(k) = G(\Omega_k)U(k) + E(k)$$



LTI system

- Frequency domain **windowed, non-periodic signal**

$$Y(k) = G(\Omega_k)U(k) + E(k) + T(\Omega_k)$$

LTI system

- Frequency domain **windowed, non-periodic signal**

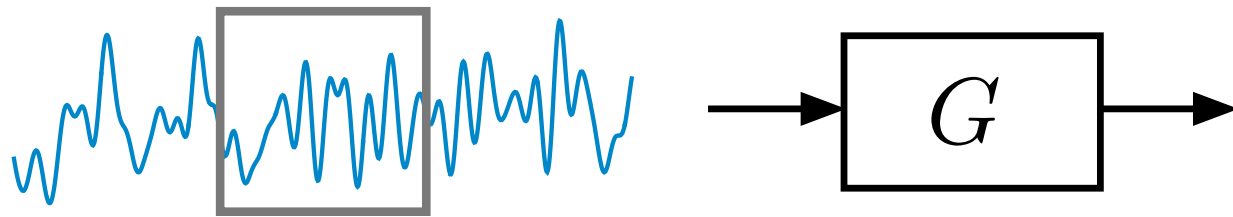
$$Y(k) = G(\Omega_k)U(k) + E(k) + T(\Omega_k)$$



LTI system

- Frequency domain **windowed, non-periodic signal**

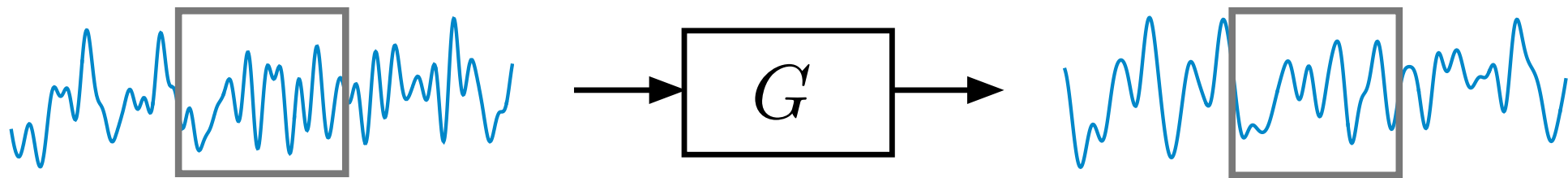
$$Y(k) = G(\Omega_k)U(k) + E(k) + T(\Omega_k)$$



LTI system

- Frequency domain **windowed, non-periodic signal**

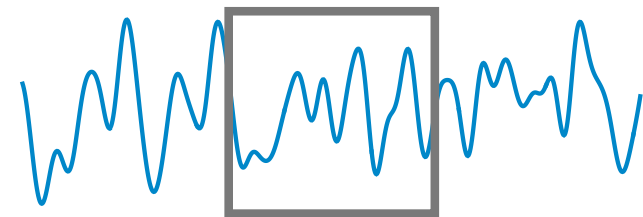
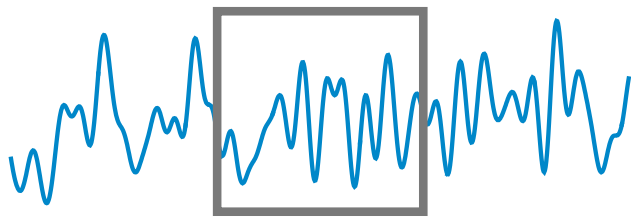
$$Y(k) = G(\Omega_k)U(k) + E(k) + T(\Omega_k)$$



LTI system

- Frequency domain **windowed, non-periodic signal**

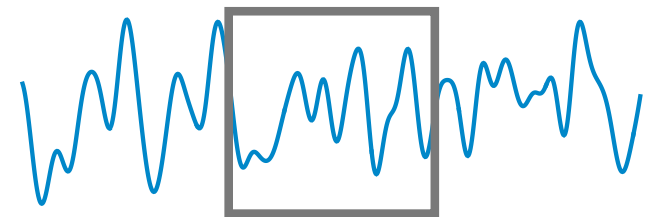
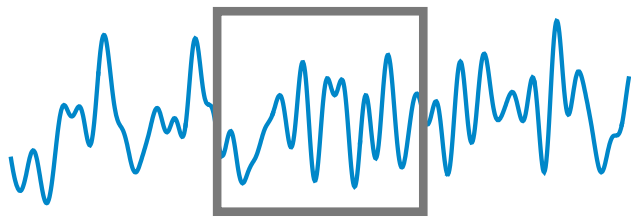
$$Y(k) = G(\Omega_k)U(k) + E(k) + T(\Omega_k)$$



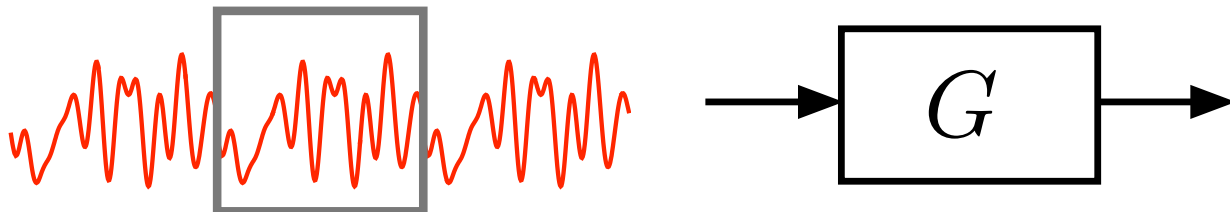
LTI system

- Frequency domain **windowed, non-periodic signal**

$$Y(k) = G(\Omega_k)U(k) + E(k) + T(\Omega_k)$$



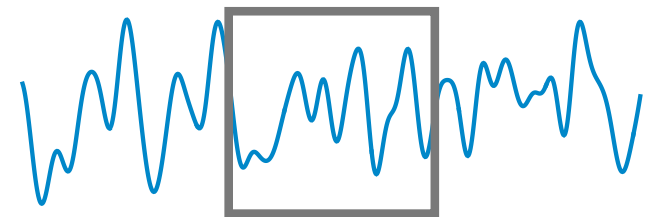
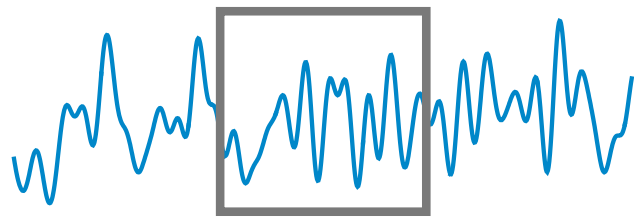
periodic



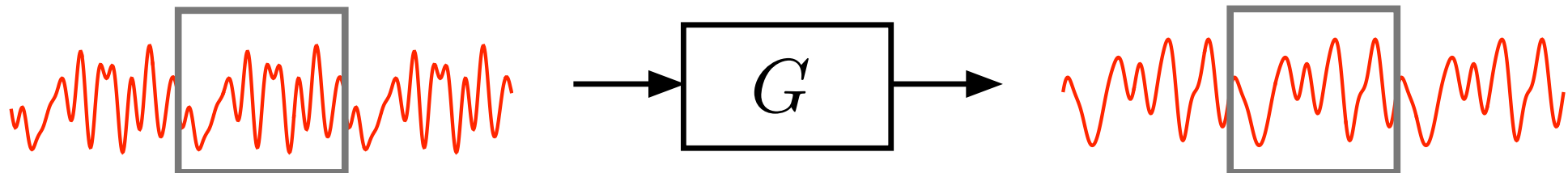
LTI system

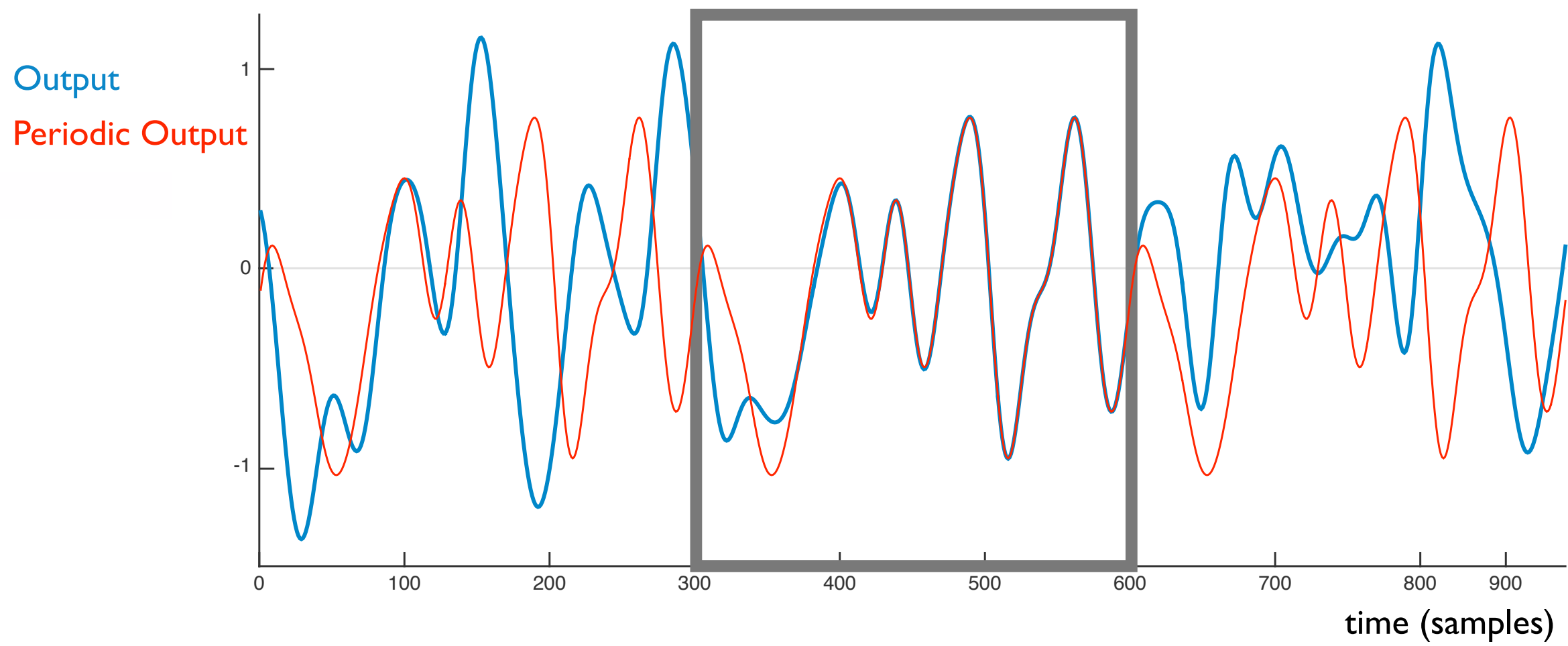
- Frequency domain **windowed, non-periodic signal**

$$Y(k) = G(\Omega_k)U(k) + E(k) + T(\Omega_k)$$

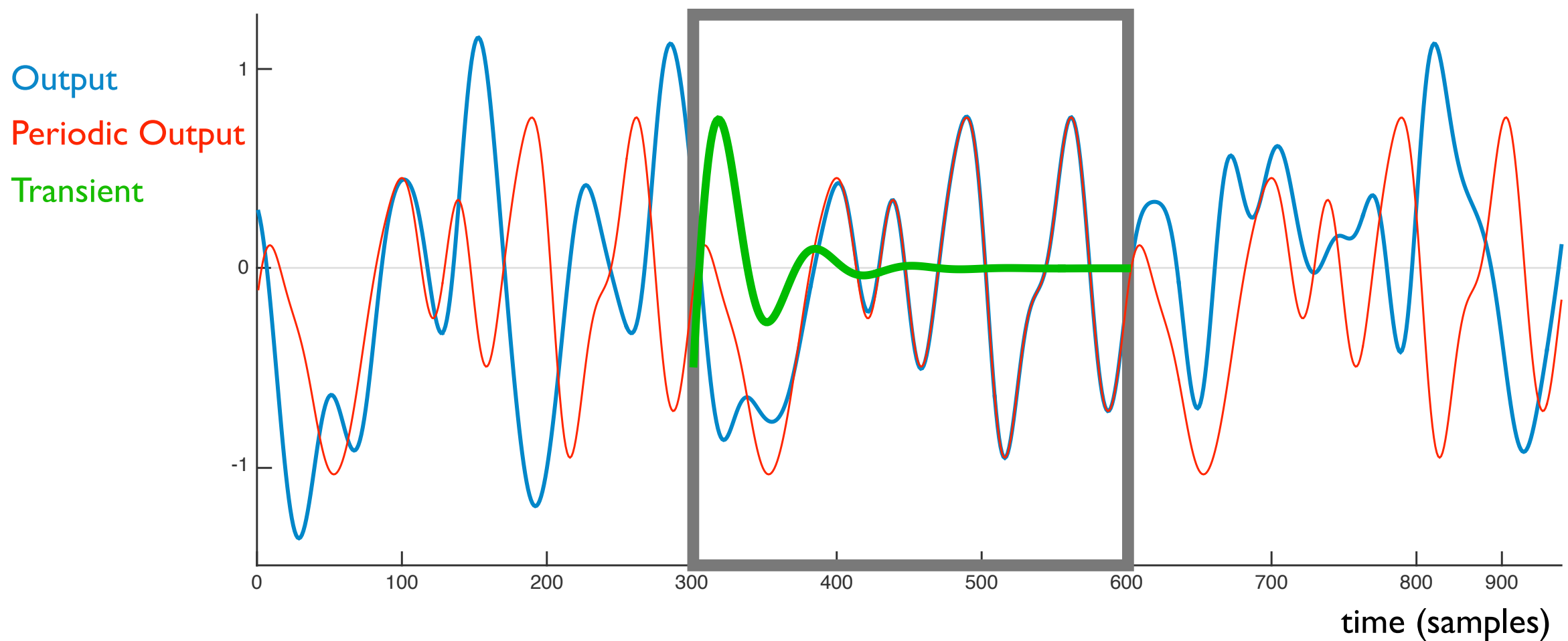


periodic





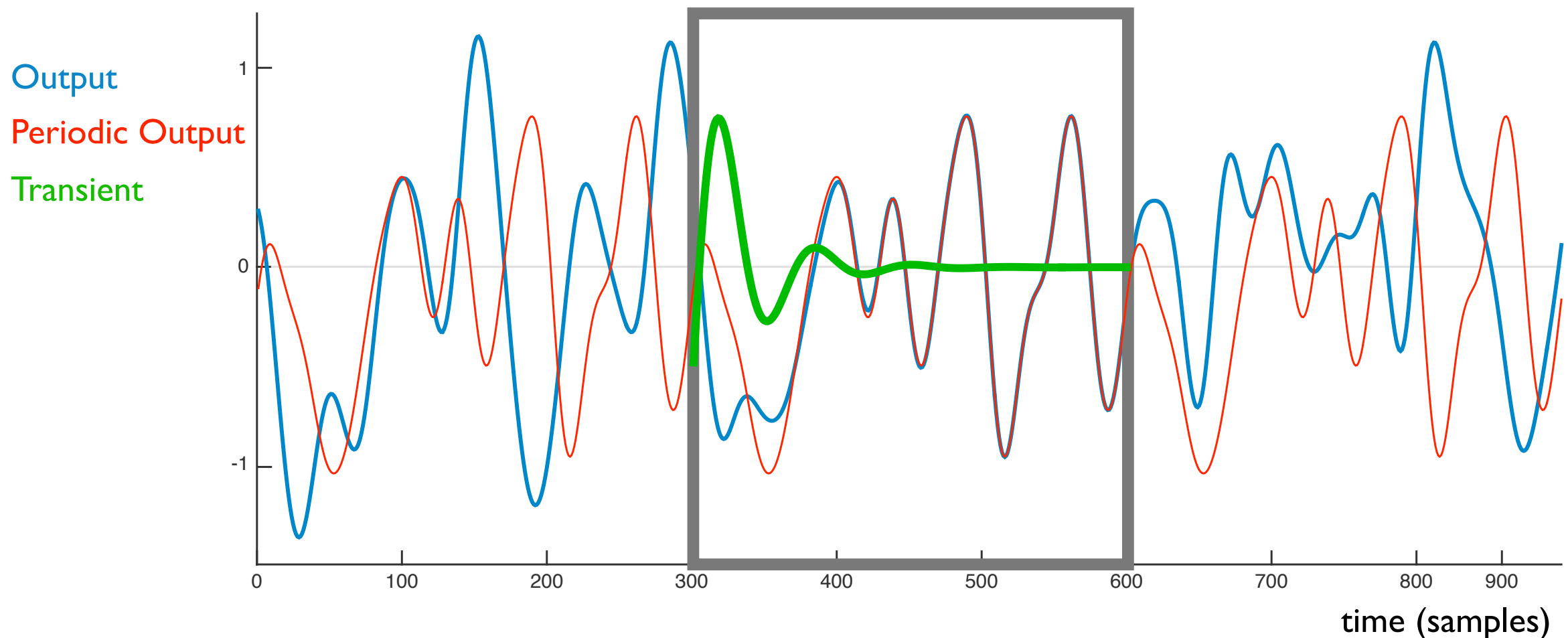
Transient term



Transient term

- Frequency domain, windowed, non-periodic excitation

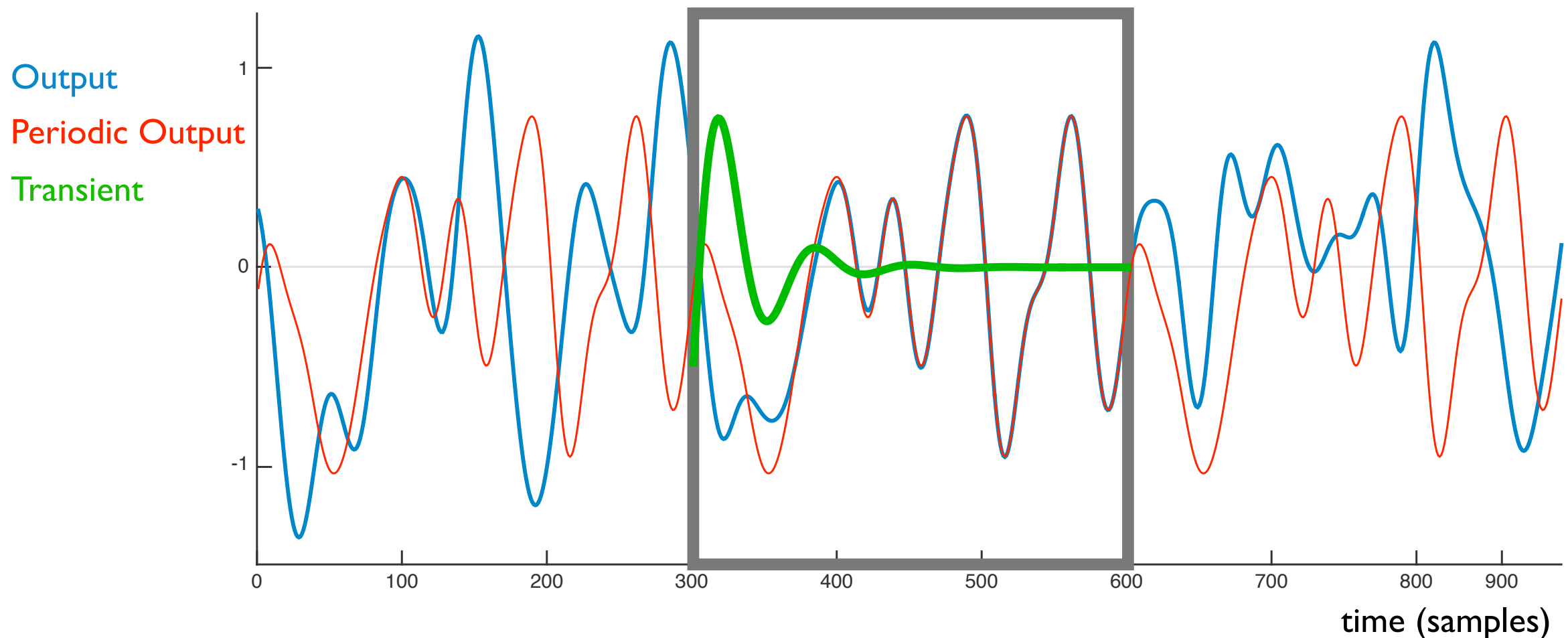
$$Y(k) = G(\Omega_k)U(k) + E(k) + T(\Omega_k)$$



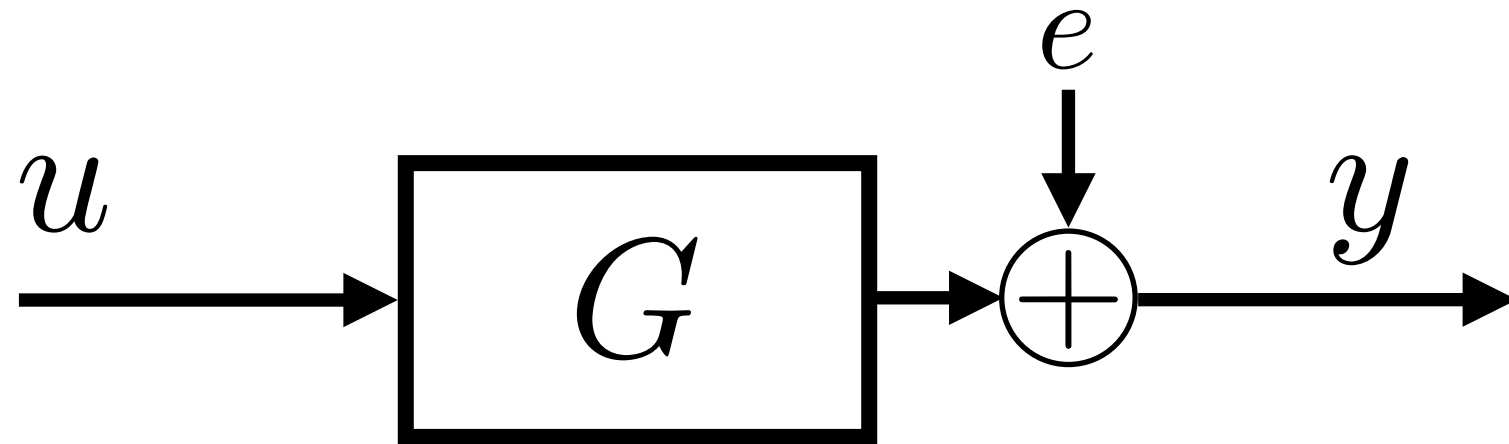
Transient term

- Frequency domain, windowed, non-periodic excitation

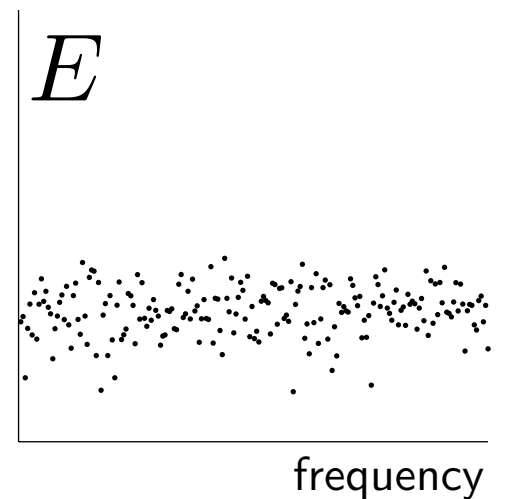
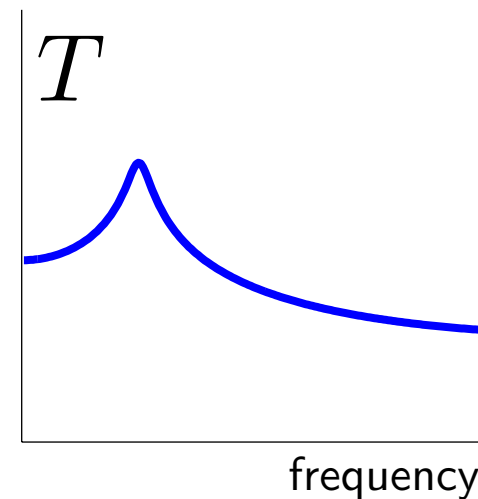
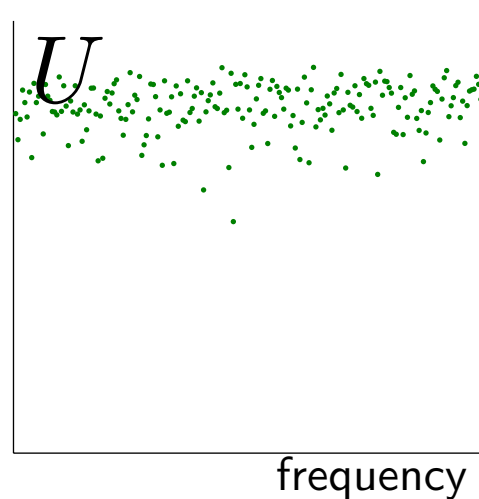
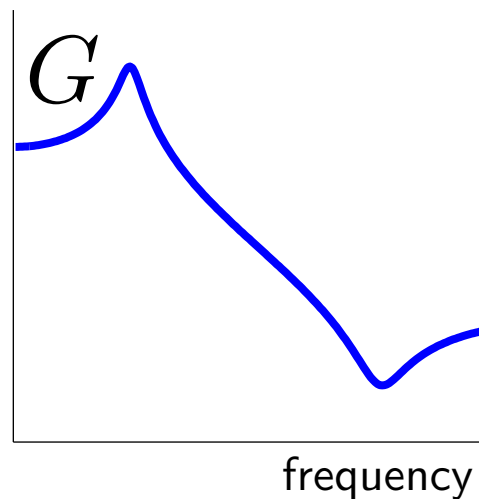
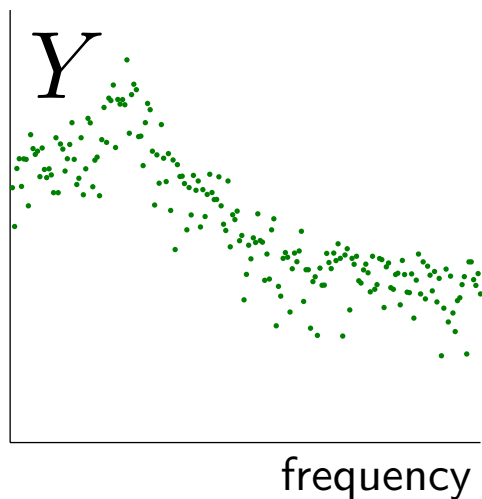
$$Y(k) = \frac{B(\Omega_k)}{A(\Omega_k)} U(k) + E(k) + \frac{I(\Omega_k)}{A(\Omega_k)}$$



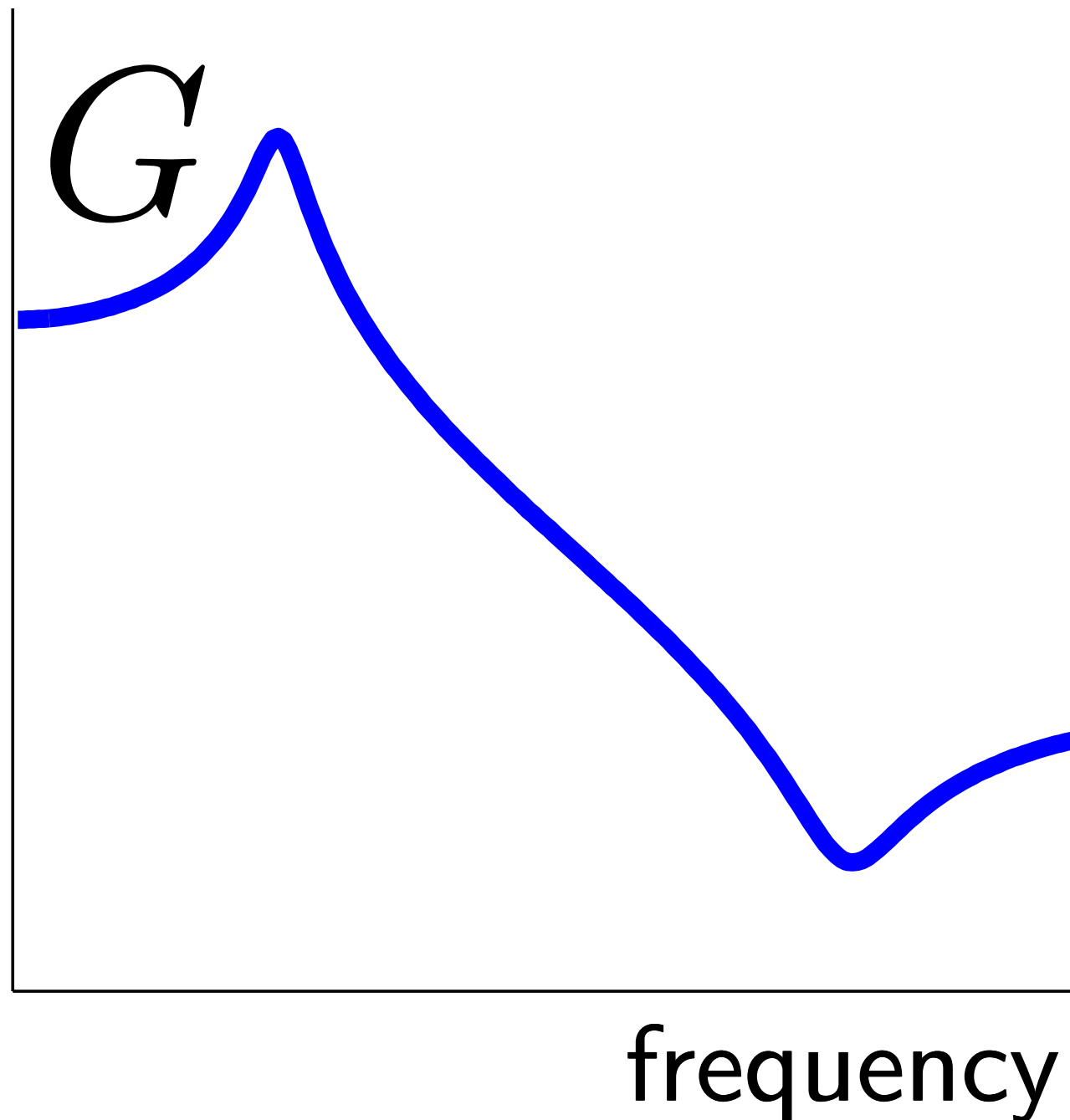
System Equation



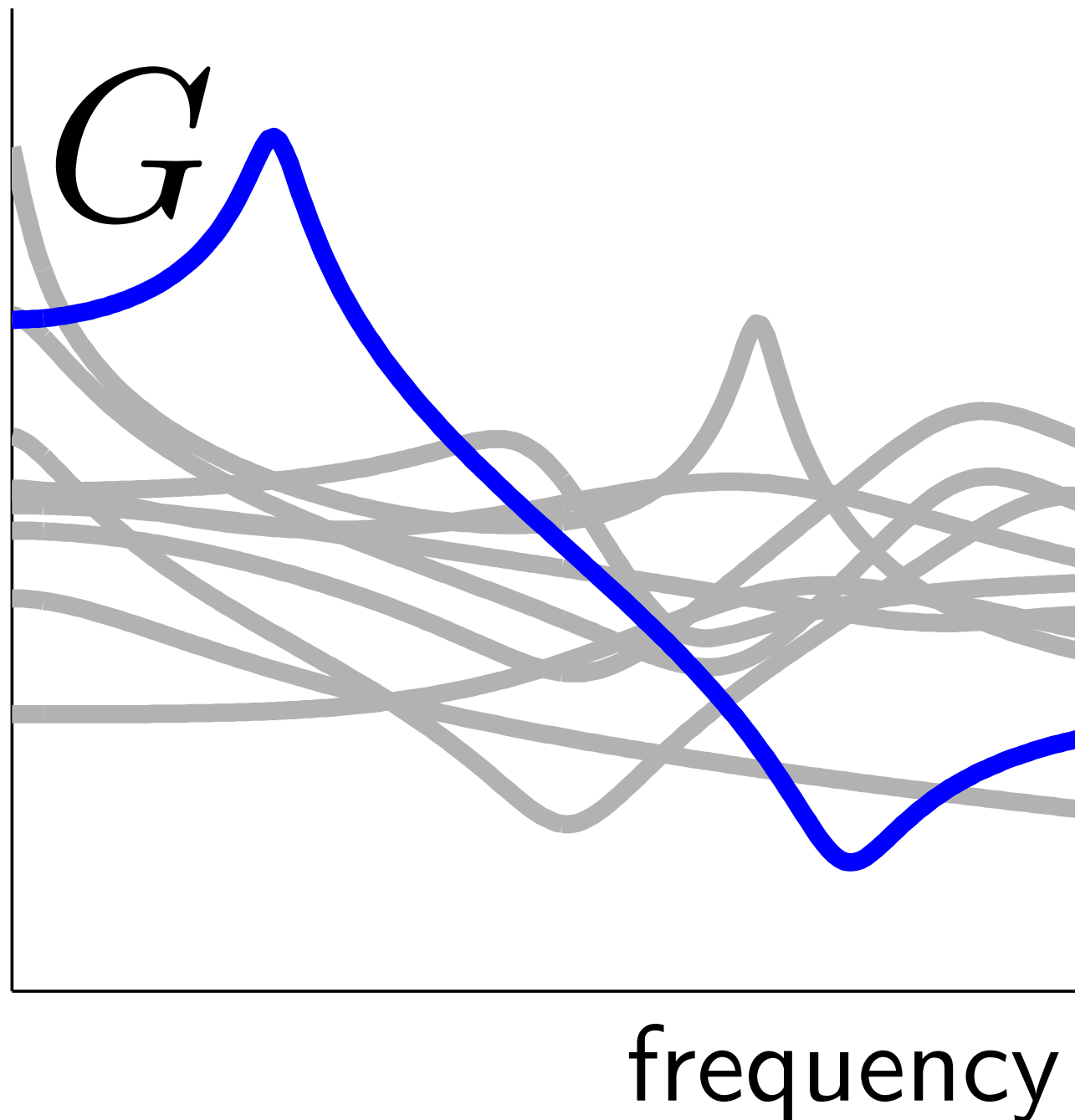
$$Y = GU + T + E$$



Model as stochastic process



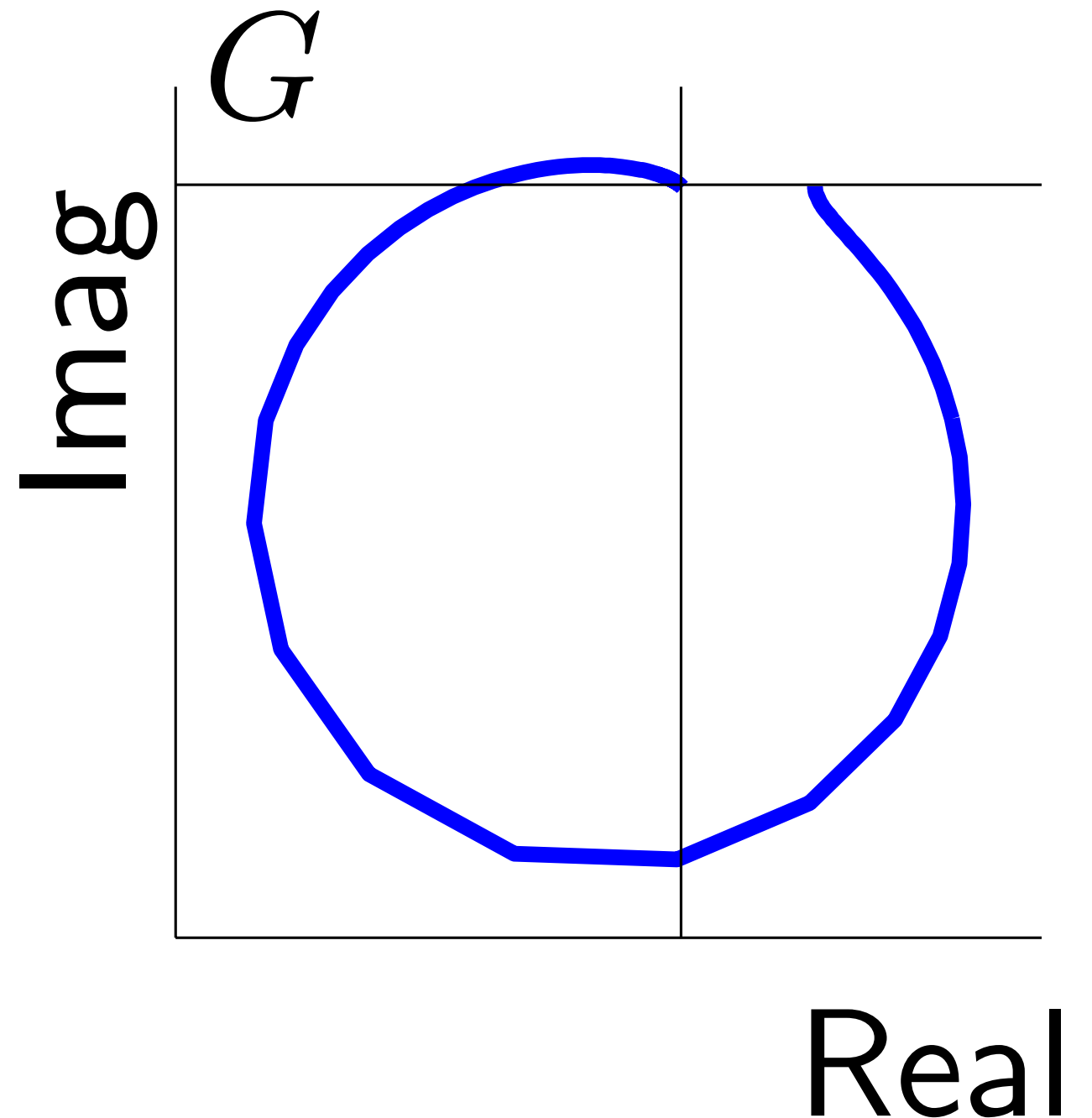
Model as stochastic process



➔ **Gaussian process**

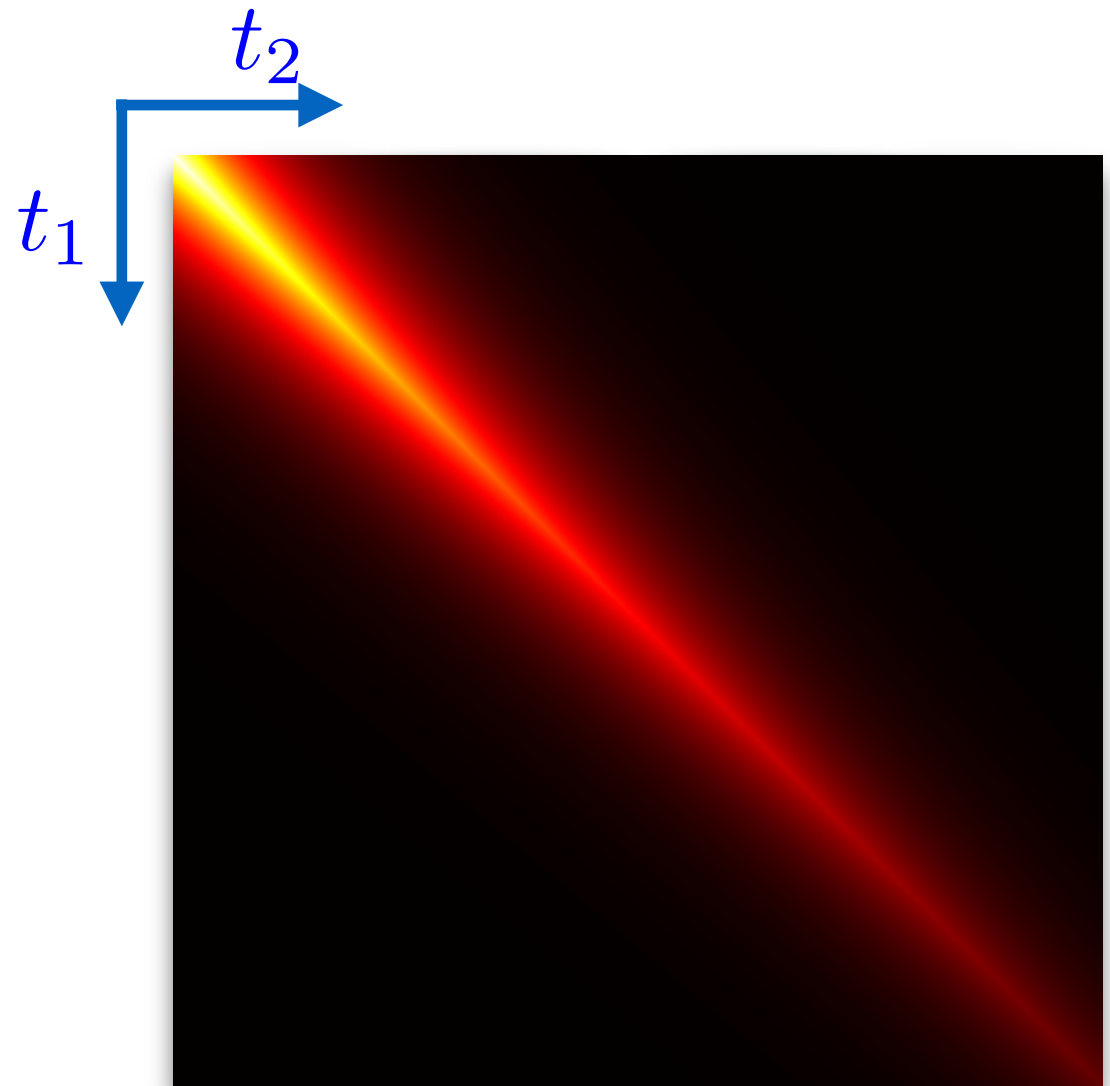
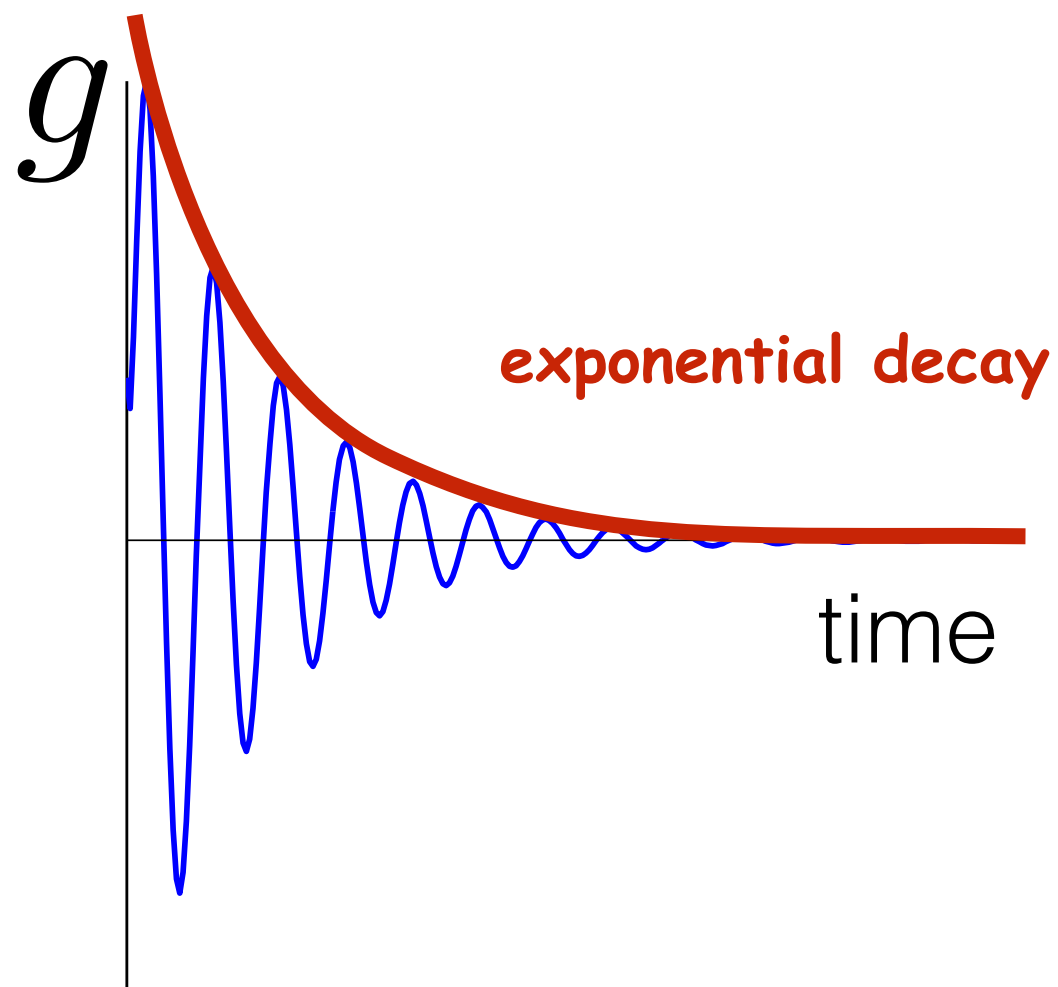
GP applied to FRF

- ✓ Smooth
- ? Complex
- ? Stability



Kernel for **STABLE** systems

- Time domain



Diagonal Correlated (DC) kernel: $K(t_1, t_2) = e^{-\alpha|t_1 - t_2|} e^{-\beta(t_1 + t_2)}$

Complex GP LTI

Time domain

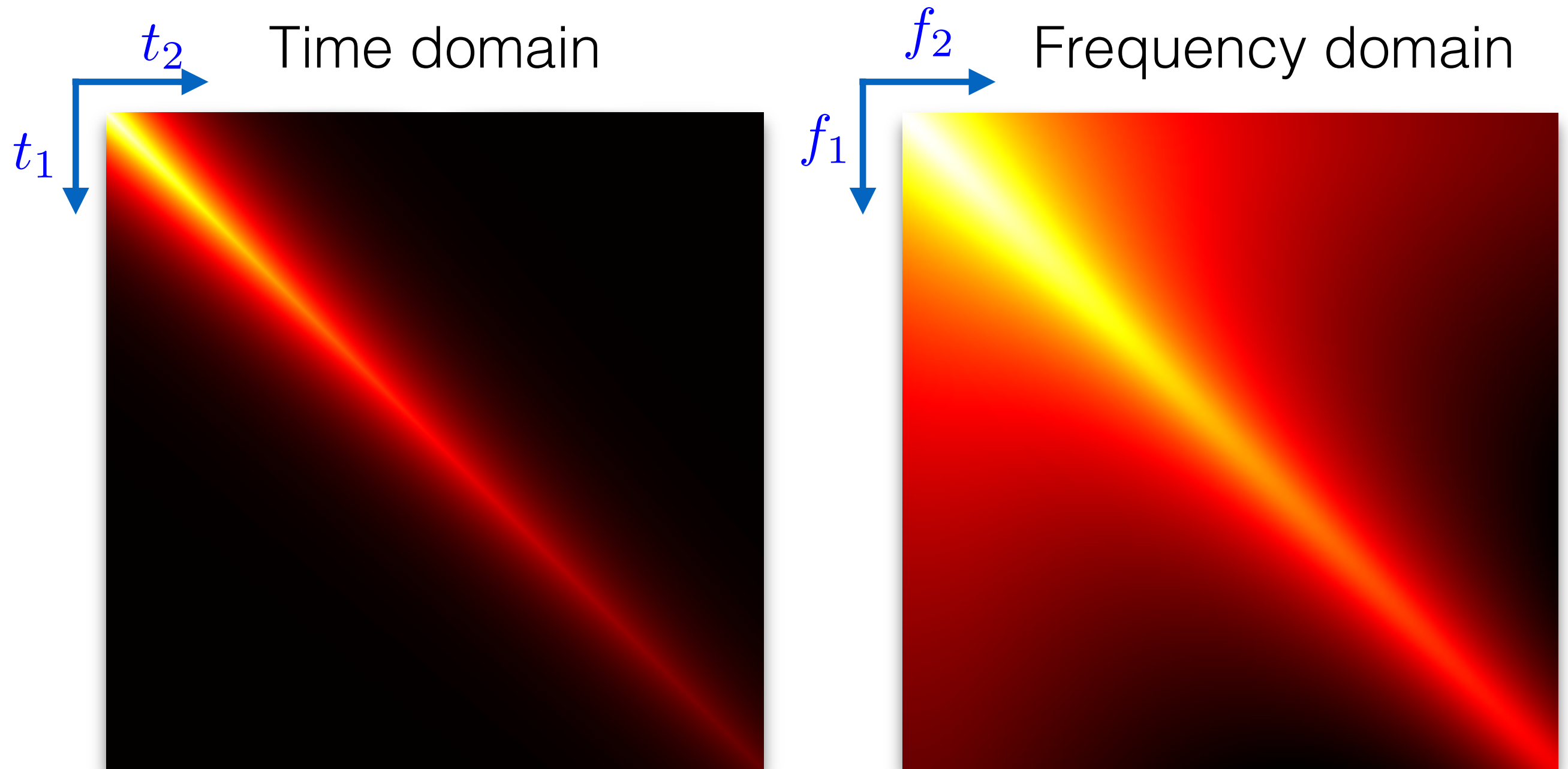
$$g \sim \mathcal{GP}(0, K_{\text{TD}}) \quad K_{\text{TD}}(t_1, t_2) = \mathbb{E} \{g(t_1)g(t_2)\}$$

Frequency domain

$$G = \mathcal{F}\{g\} \quad G \sim \mathcal{GP}(0, K_{\text{FD}})$$

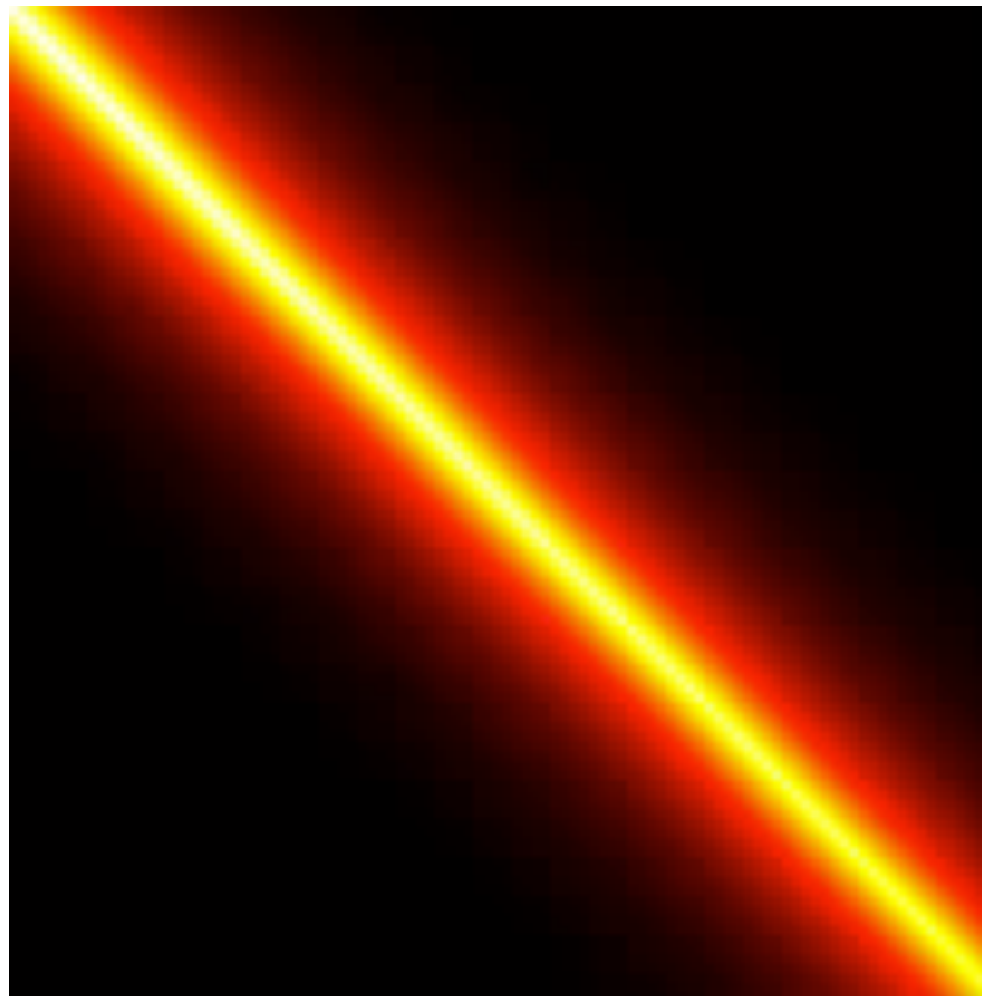
$$\begin{aligned} K_{\text{FD}}(\Omega_1, \Omega_2) &= \mathbb{E} \left\{ G(\Omega_1) \overline{G(\Omega_2)} \right\} \\ &= \mathcal{F}_{t_1} \left\{ \overline{\mathcal{F}_{t_2} \left\{ \mathbb{E} \{g(t_1)g(t_2)\} \right\}} \mid_{\Omega_2} \right\} \mid_{\Omega_1} \\ &= \mathcal{F}_{t_1} \left\{ \overline{\mathcal{F}_{t_2} \{K_{\text{TD}}(t_1, t_2)\}} \mid_{\Omega_2} \right\} \mid_{\Omega_1} \end{aligned}$$

Kernel for **STABLE** systems

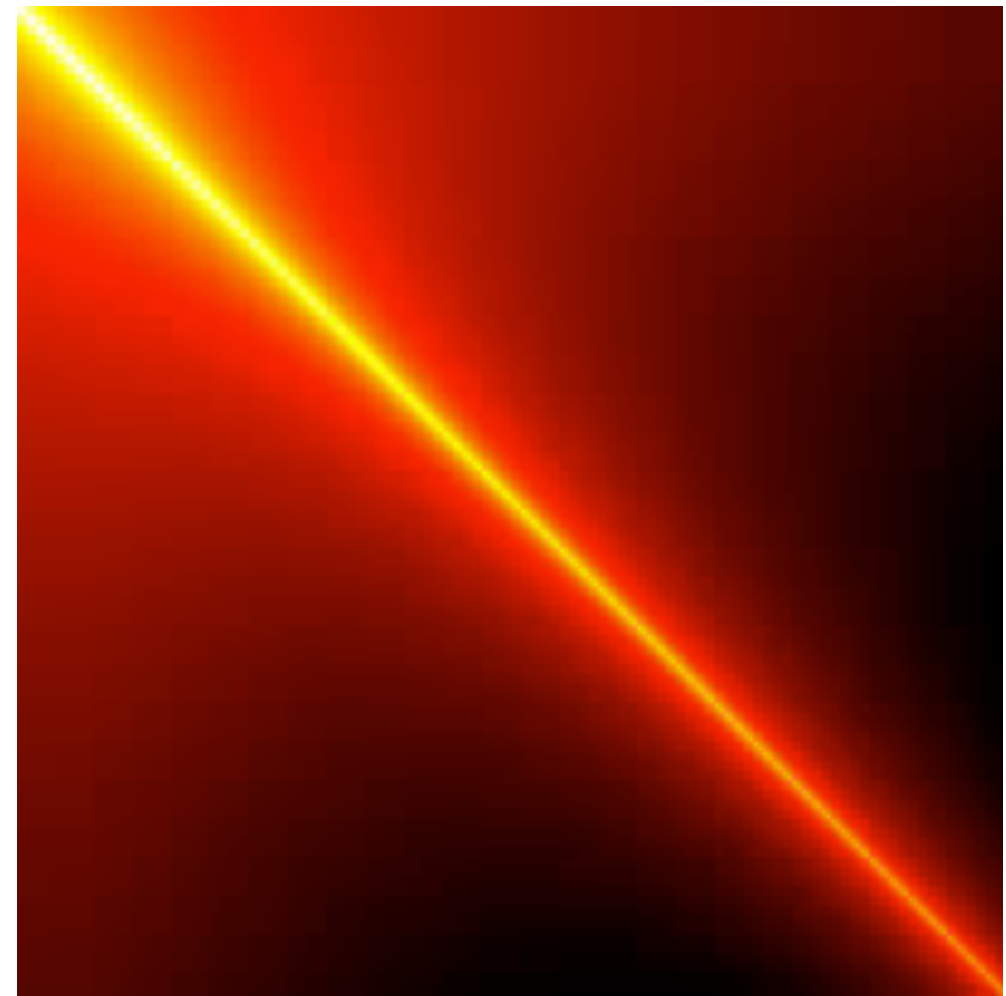


Decay Rate

Time domain



Frequency domain



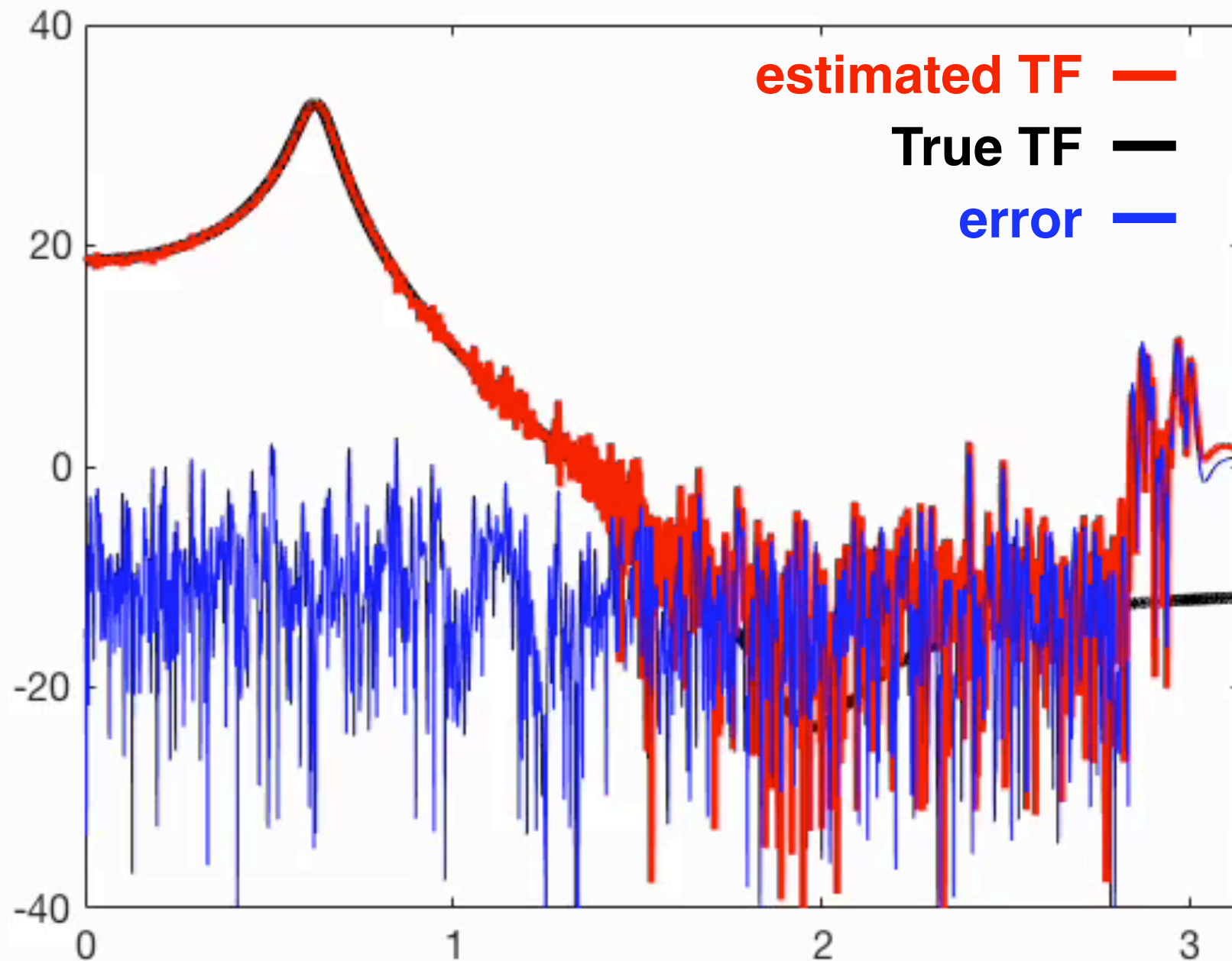
Decay Rate

Long

Short

Scan of decay rate

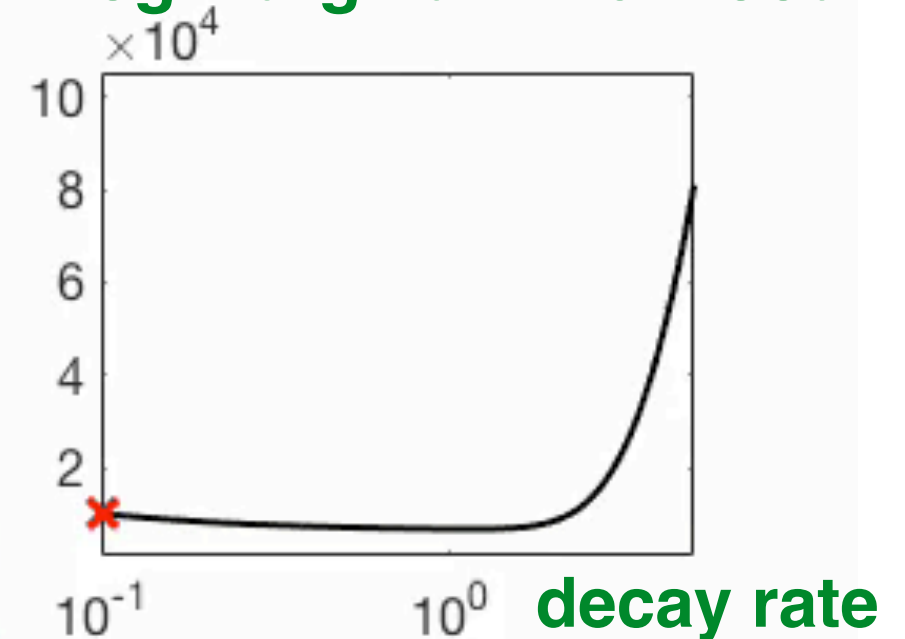
amplitude (dB)



estimated TF —
True TF —
error —

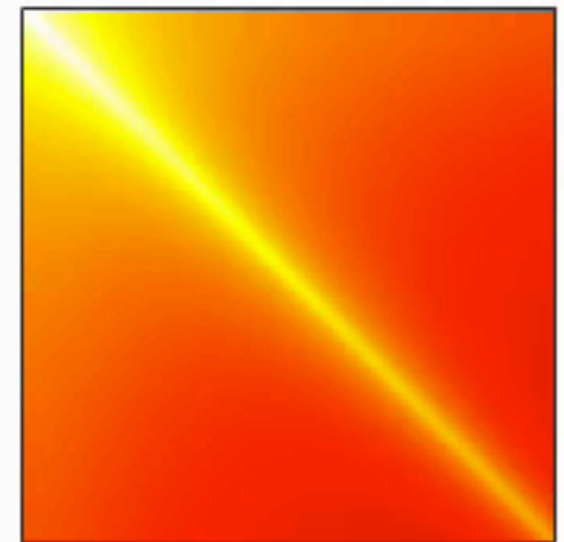
frequency (Hz)

- log Marginal Likelihood



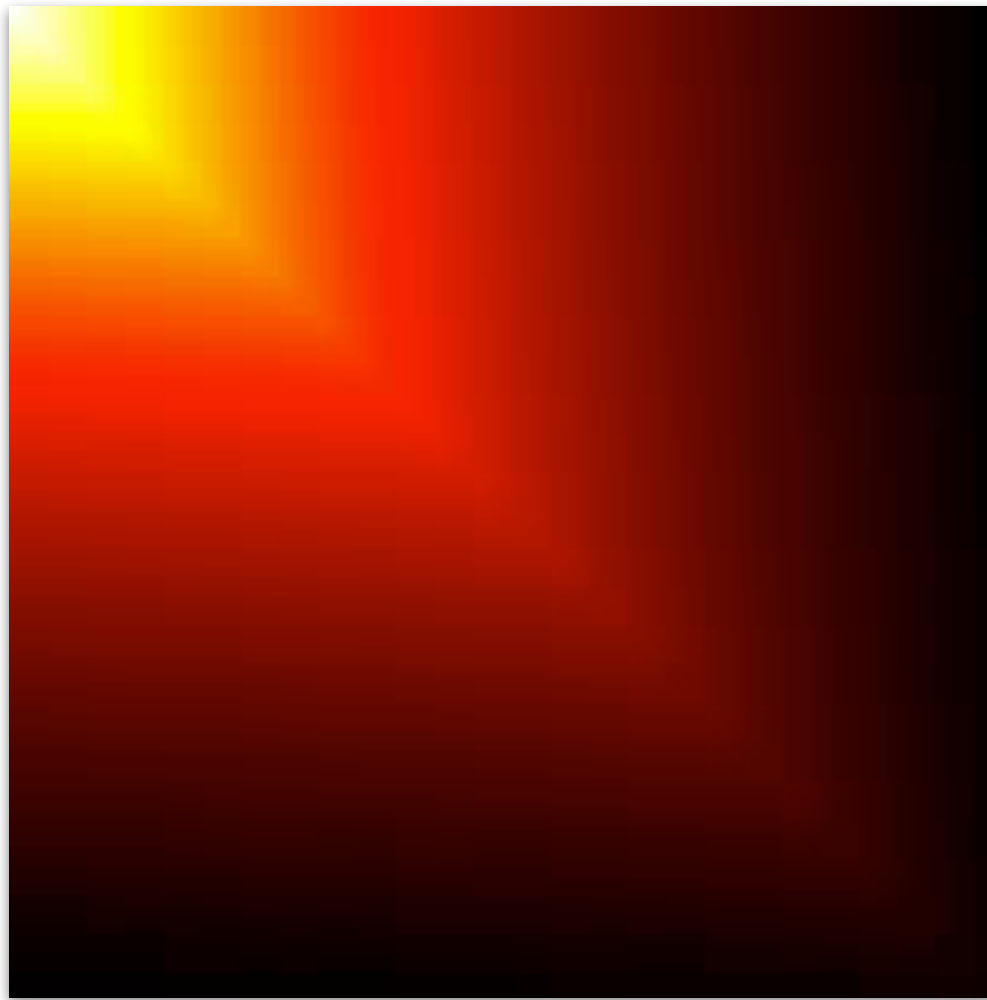
decay rate

Kernel

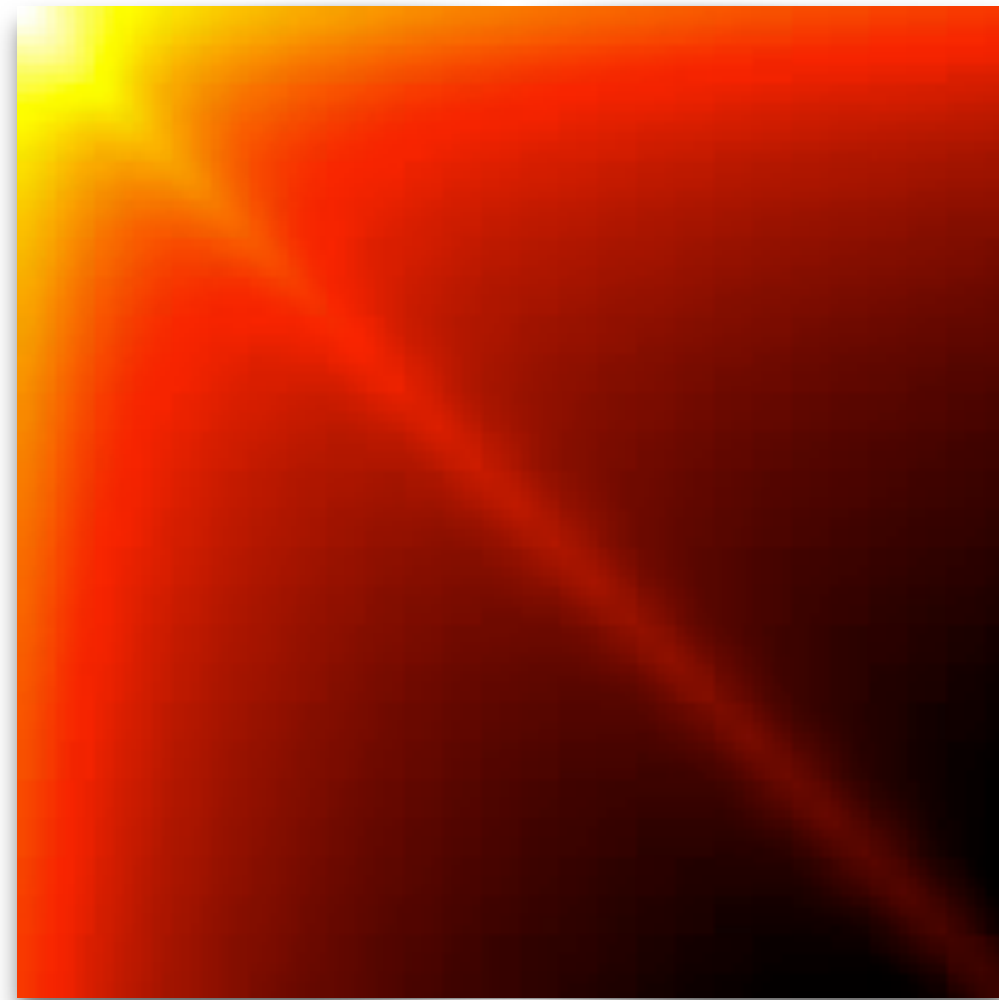


Time correlation

Time domain



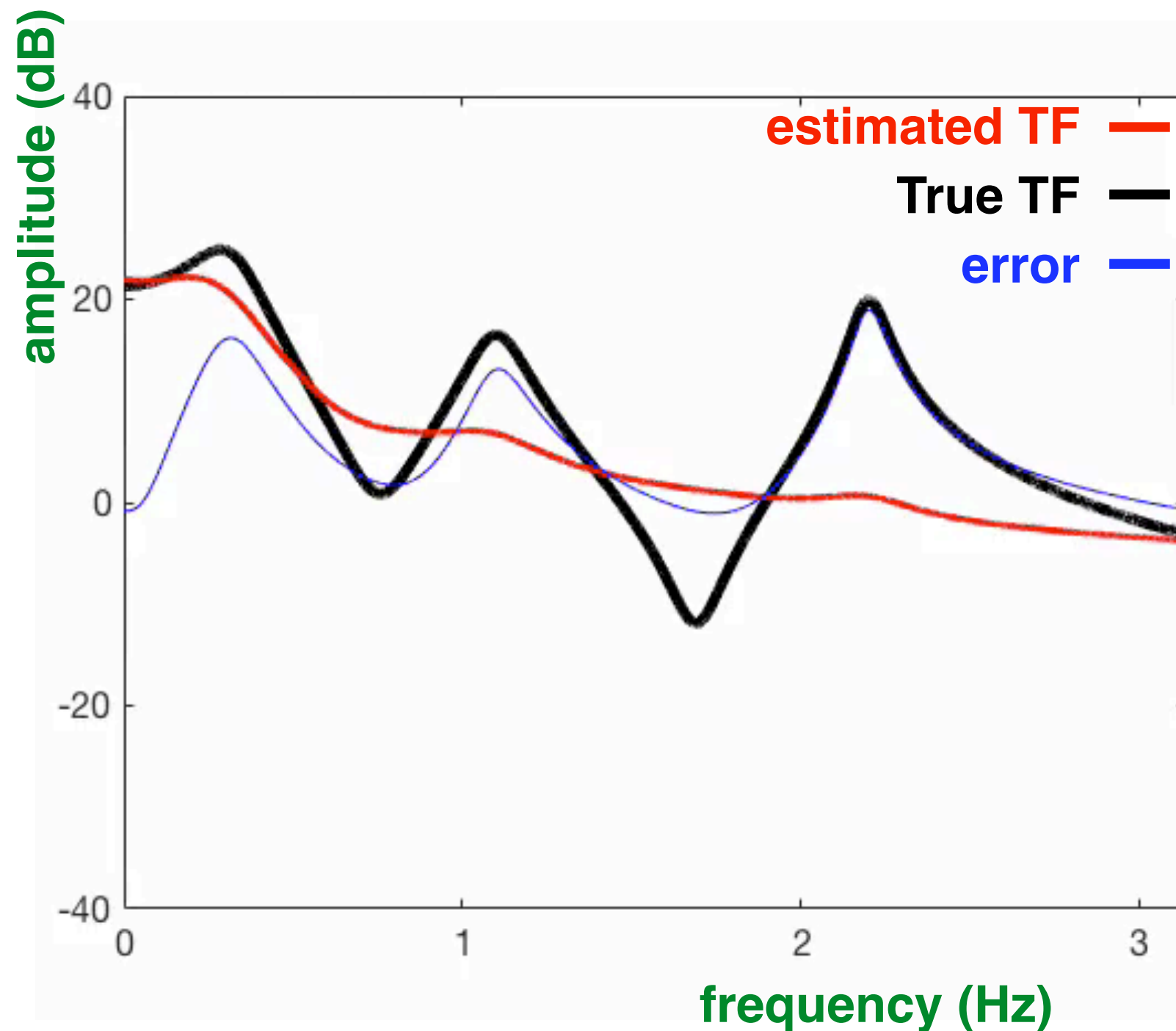
Frequency domain



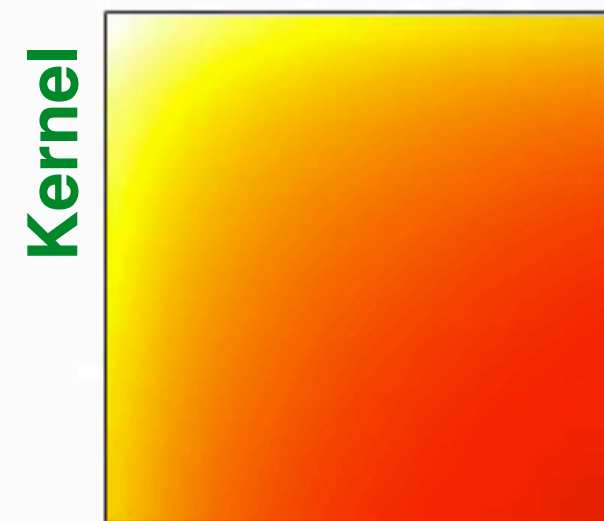
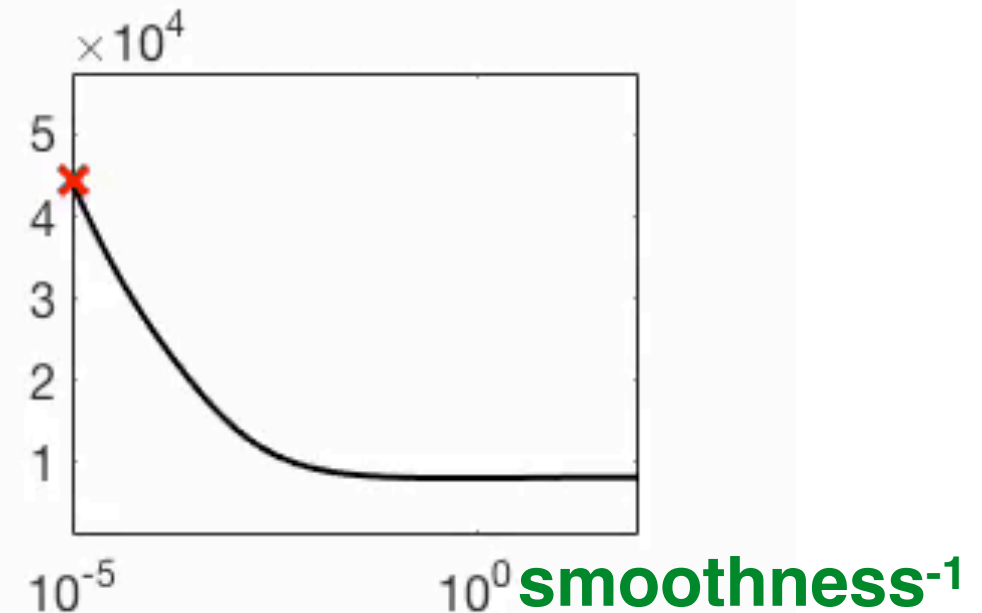
Time correlation



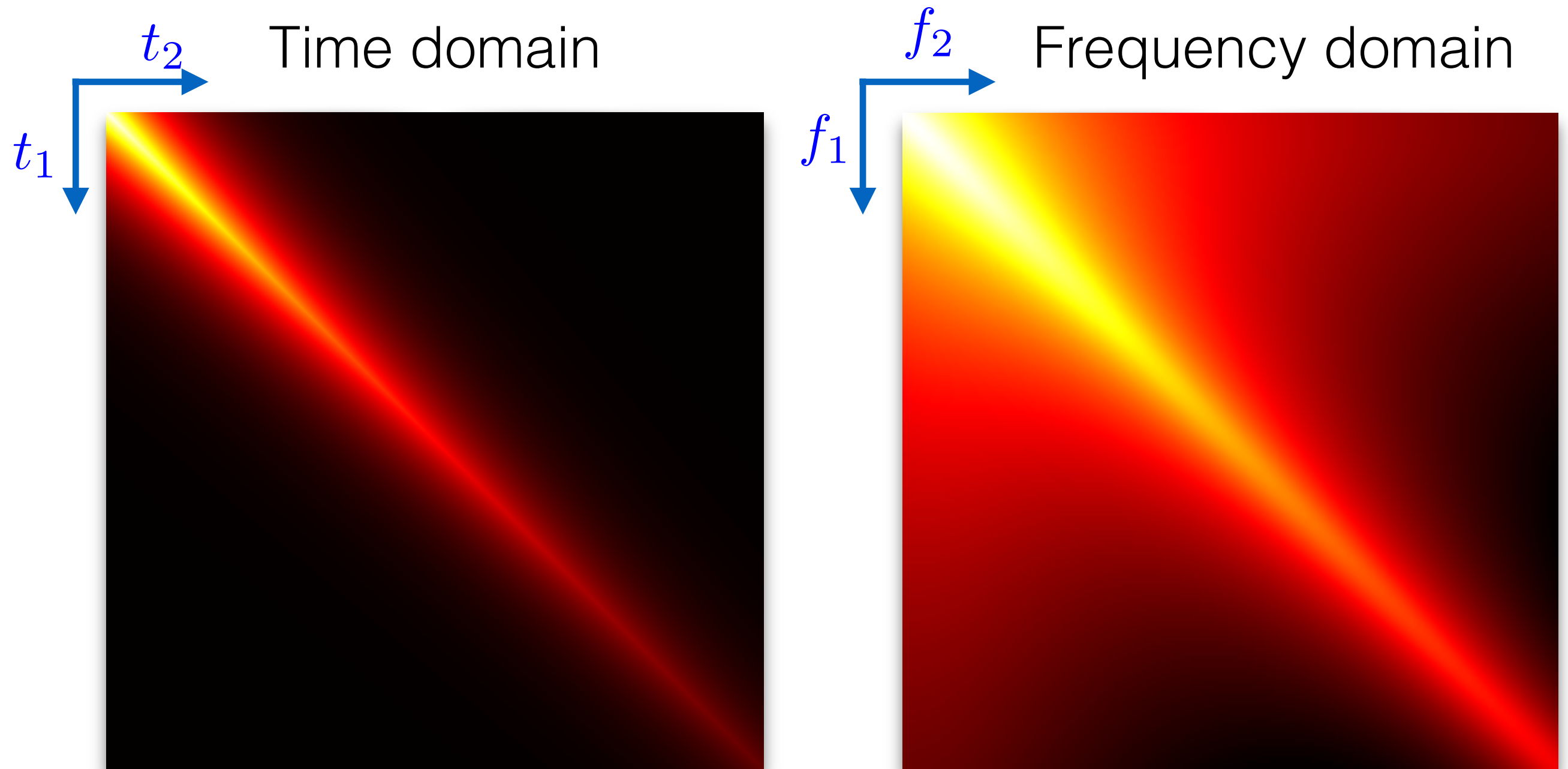
Scan of smoothness



- log Marginal Likelihood



Kernel for **STABLE** systems



Complex Gaussian process

- Naive approach

$$p(G) = \frac{1}{\pi^n \cdot |K_{\text{FD}}|} \cdot e^{-G^{\text{H}} K_{\text{FD}}^{-1} G}$$

- Real / Imaginary

$$p \begin{pmatrix} \Re G \\ \Im G \end{pmatrix} = \frac{1}{\sqrt{(2\pi)^n |K_{\text{re}}|}} \cdot e^{-\frac{1}{2} \begin{bmatrix} \Re G \\ \Im G \end{bmatrix}^{\text{H}} \begin{bmatrix} K_{\Re\Re} & K_{\Re\Im} \\ K_{\Im\Re} & K_{\Im\Im} \end{bmatrix}^{-1} \begin{bmatrix} \Re G \\ \Im G \end{bmatrix}}$$

- But then:

$$\begin{bmatrix} G \\ \overline{G} \end{bmatrix} = \begin{bmatrix} I & jI \\ I & -jI \end{bmatrix} \begin{bmatrix} \Re G \\ \Im G \end{bmatrix}$$

Complex Gaussian process

- Naive approach

$$p(G) = \frac{1}{\pi^n \cdot |K_{\text{FD}}|} \cdot e^{-G^{\text{H}} K_{\text{FD}}^{-1} G}$$

- Real / Imaginary

$$p \begin{pmatrix} \Re G \\ \Im G \end{pmatrix} = \frac{1}{\sqrt{(2\pi)^n |K_{\text{re}}|}} \cdot e^{-\frac{1}{2} \underbrace{\begin{bmatrix} \Re G \\ \Im G \end{bmatrix}^{\text{H}}}_{M^{-\text{H}} M^{\text{H}}} \underbrace{\begin{bmatrix} K_{\Re\Re} & K_{\Re\Im} \\ K_{\Im\Re} & K_{\Im\Im} \end{bmatrix}^{-1}}_{M M^{-1}} \begin{bmatrix} \Re G \\ \Im G \end{bmatrix}}$$

- But then:

$$\begin{bmatrix} G \\ \overline{G} \end{bmatrix} = \underbrace{M^{-1}}_{M^{-\text{H}} M^{\text{H}}} \begin{bmatrix} \Re G \\ \Im G \end{bmatrix}$$

Complex Gaussian process

- Naive approach

$$p(G) = \frac{1}{\pi^n \cdot |K_{\text{FD}}|} \cdot e^{-G^{\text{H}} K_{\text{FD}}^{-1} G}$$

- Real / Imaginary

$$p \left(\begin{array}{c} \Re G \\ \Im G \end{array} \right) = \frac{1}{\sqrt{(2\pi)^n |K_{\text{re}}|}} \cdot e^{-\frac{1}{2} \begin{bmatrix} G \\ \overline{G} \end{bmatrix}^{\text{H}} \begin{bmatrix} K_{\text{FD}} & \textcolor{blue}{C}_{\text{FD}} \\ \overline{\textcolor{blue}{C}_{\text{FD}}} & \overline{K_{\text{FD}}} \end{bmatrix}^{-1} \begin{bmatrix} G \\ \overline{G} \end{bmatrix}}$$

- But then:

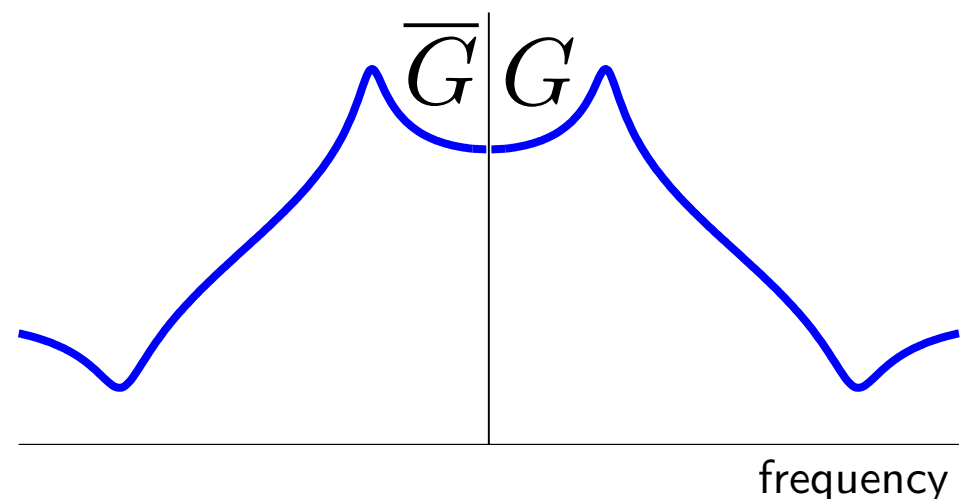
$$\begin{bmatrix} G \\ \overline{G} \end{bmatrix} = \textcolor{blue}{M}^{-1} \begin{bmatrix} \Re G \\ \Im G \end{bmatrix}$$

Complex Gaussian process

$$p\left(\frac{G}{\overline{G}}\right) = \frac{1}{\pi^n \sqrt{|\Gamma|}} \cdot e^{-\frac{1}{2} \begin{bmatrix} G \\ \overline{G} \end{bmatrix}^H \underbrace{\begin{bmatrix} K_{\text{FD}} & C_{\text{FD}} \\ \overline{C_{\text{FD}}} & \overline{K_{\text{FD}}} \end{bmatrix}}_{\Gamma} \begin{bmatrix} G \\ \overline{G} \end{bmatrix}}$$

$$K_{\text{FD}}(j\omega_1, j\omega_2) = \mathbb{E} \left\{ G(j\omega_1) \overline{G(j\omega_2)} \right\}$$

$$\begin{aligned} C_{\text{FD}}(j\omega_1, j\omega_2) &= \mathbb{E} \left\{ G(j\omega_1) G(j\omega_2) \right\} \\ &= K_{\text{FD}}(j\omega_1, -j\omega_2) \end{aligned}$$

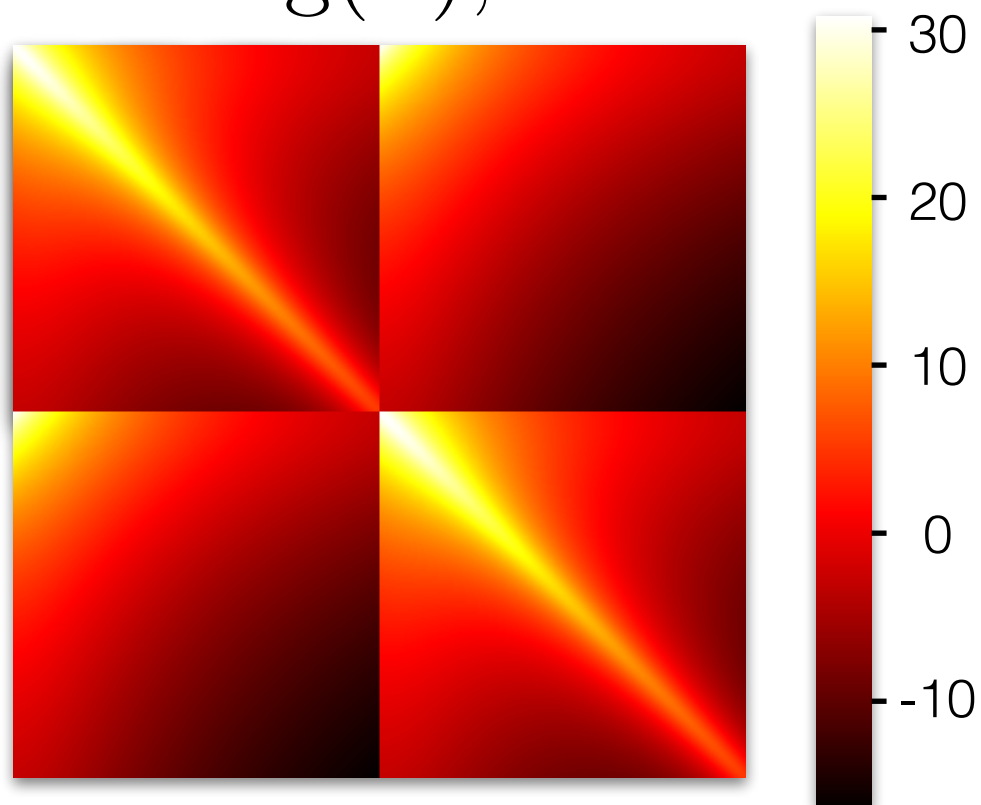


Complex Kernels

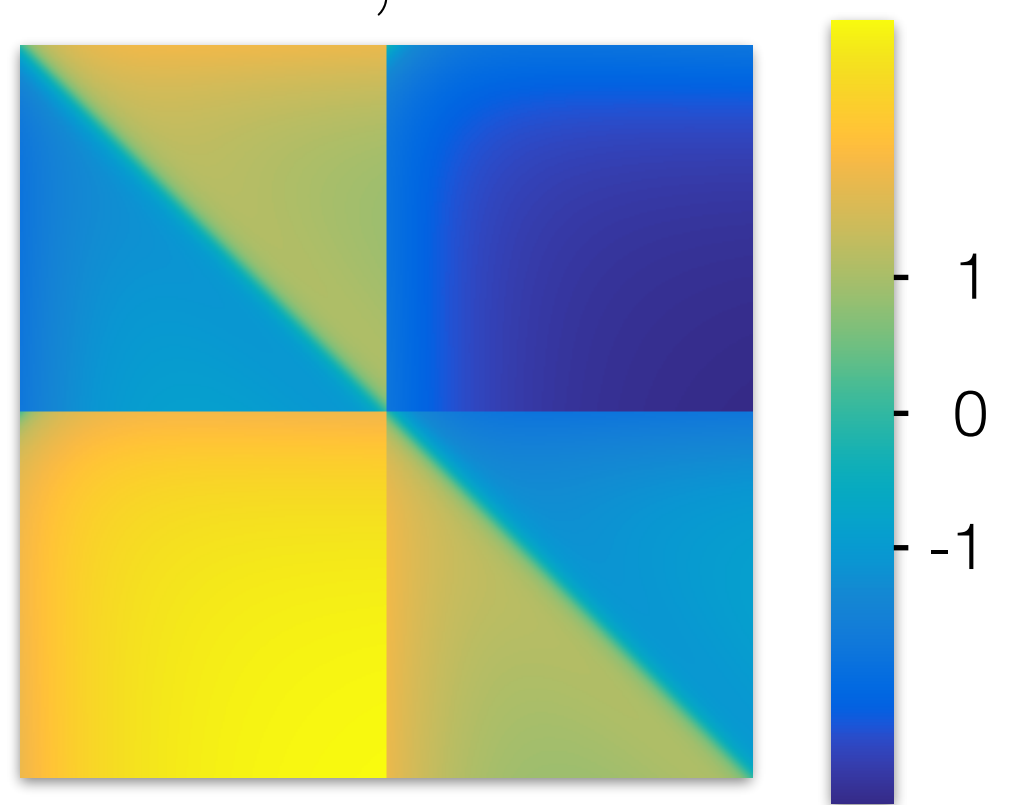
$$\Gamma \triangleq \text{cov} \begin{bmatrix} G \\ \overline{G} \end{bmatrix} = \mathbb{E} \begin{bmatrix} GG^H & GG^T \\ \overline{G}G^H & \overline{G}G^T \end{bmatrix} = \begin{bmatrix} K_{\text{FD}} & C_{\text{FD}} \\ \overline{C_{\text{FD}}} & \overline{K_{\text{FD}}} \end{bmatrix}$$

Mag(Γ), dB

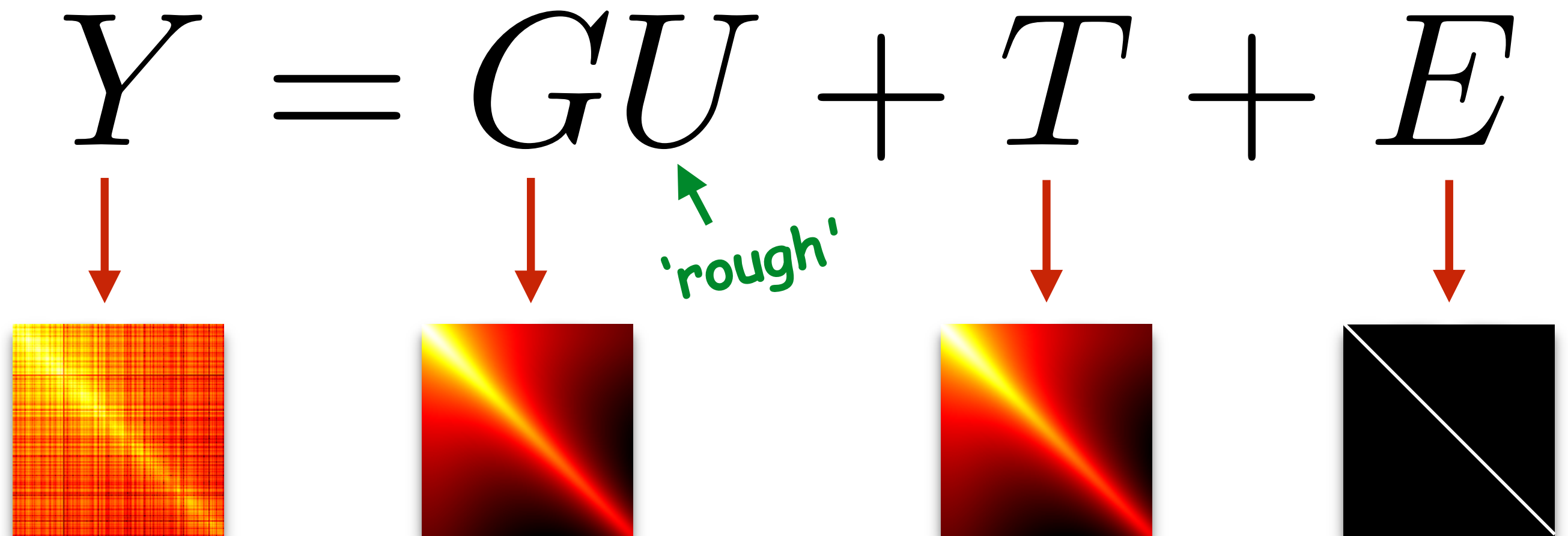
K_{FD}



$\angle \Gamma$, rad

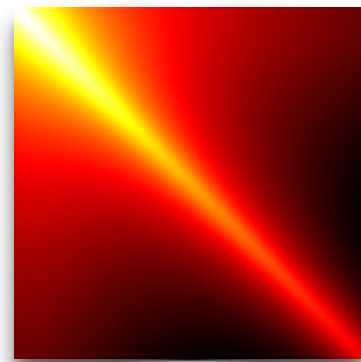
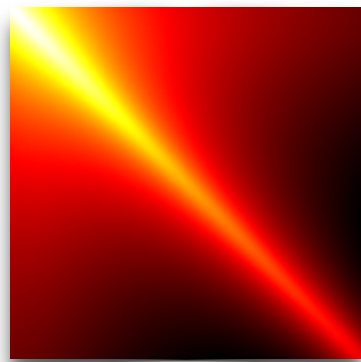
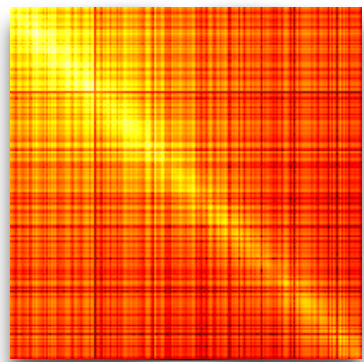


Output spectrum as Gaussian process



Estimating the FRF

$$Y = GU + T + E$$



$$\hat{G} = \begin{matrix} \text{[Heatmap of } U^H \text{]} \end{matrix} U_d^H \begin{matrix} \text{[Heatmap of } Y \text{]} \end{matrix}^{-1}$$

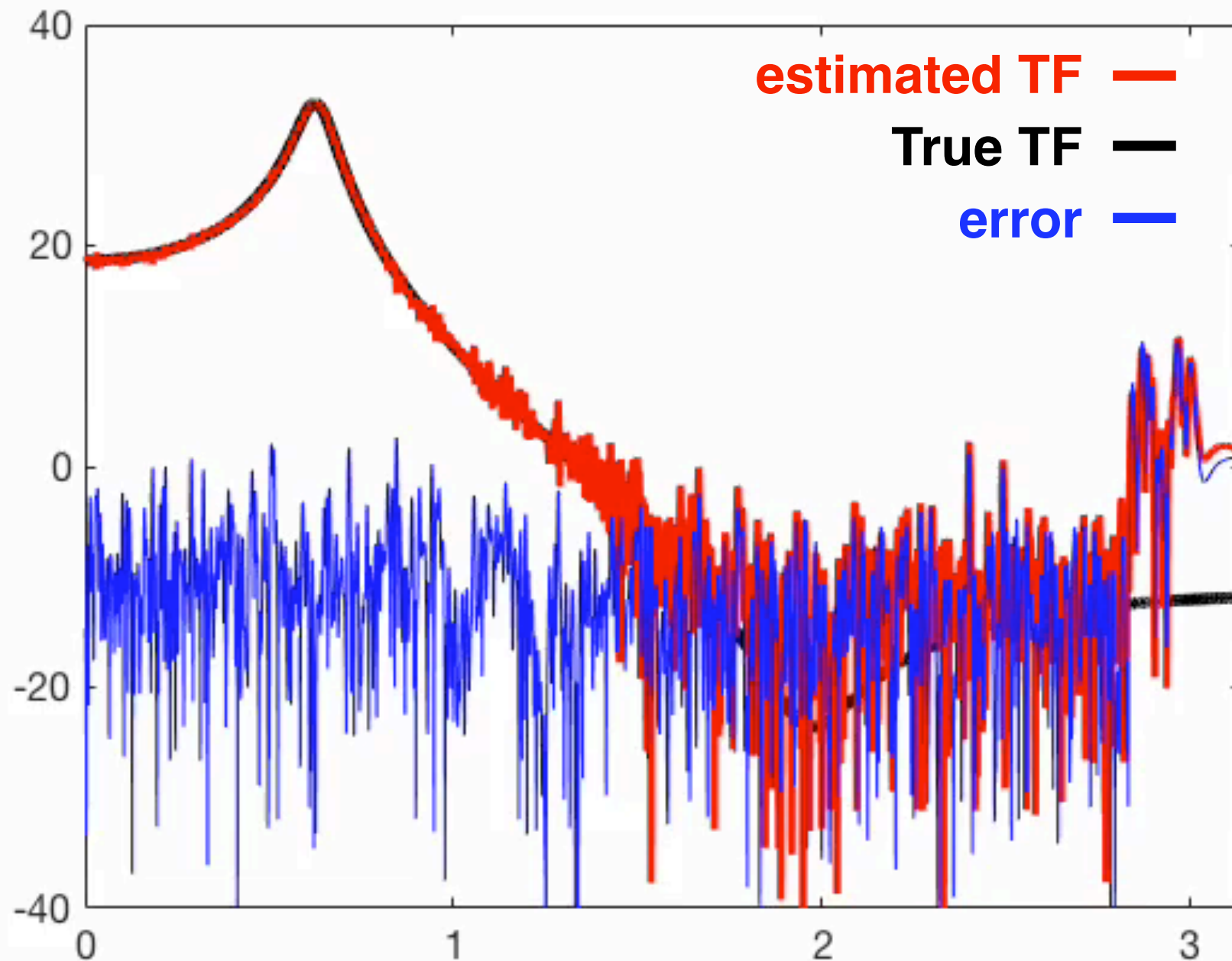
Kernel Tuning

$$Y \sim \mathcal{GP}(0, \text{Heatmap}(\theta))$$

$$\underbrace{\log p(Y|\theta)}_{\substack{\text{maximise} \\ \text{w.r.t. } \theta}} \propto \underbrace{-Y^H \text{Heatmap}(\theta)^{-1} Y}_{\text{data fit}} - \underbrace{\log |\text{Heatmap}(\theta)|}_{\text{complexity}}$$

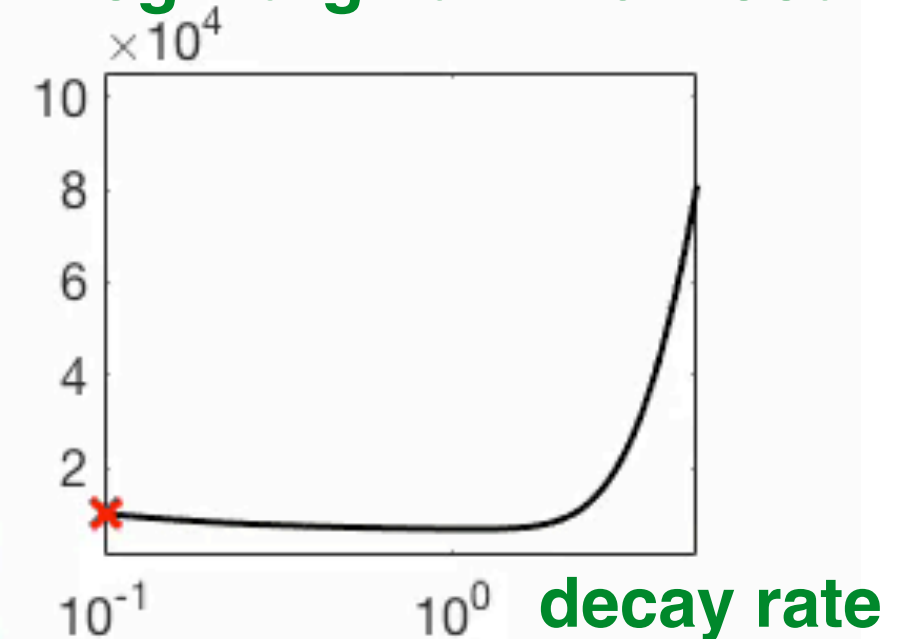
Complexity tuning

amplitude (dB)



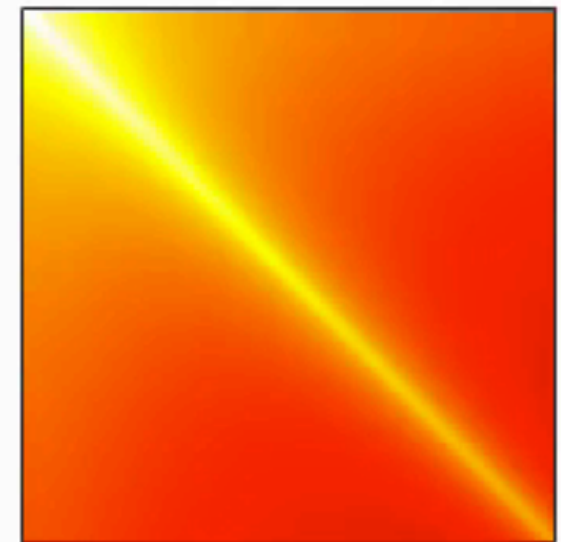
frequency (Hz)

- log Marginal Likelihood



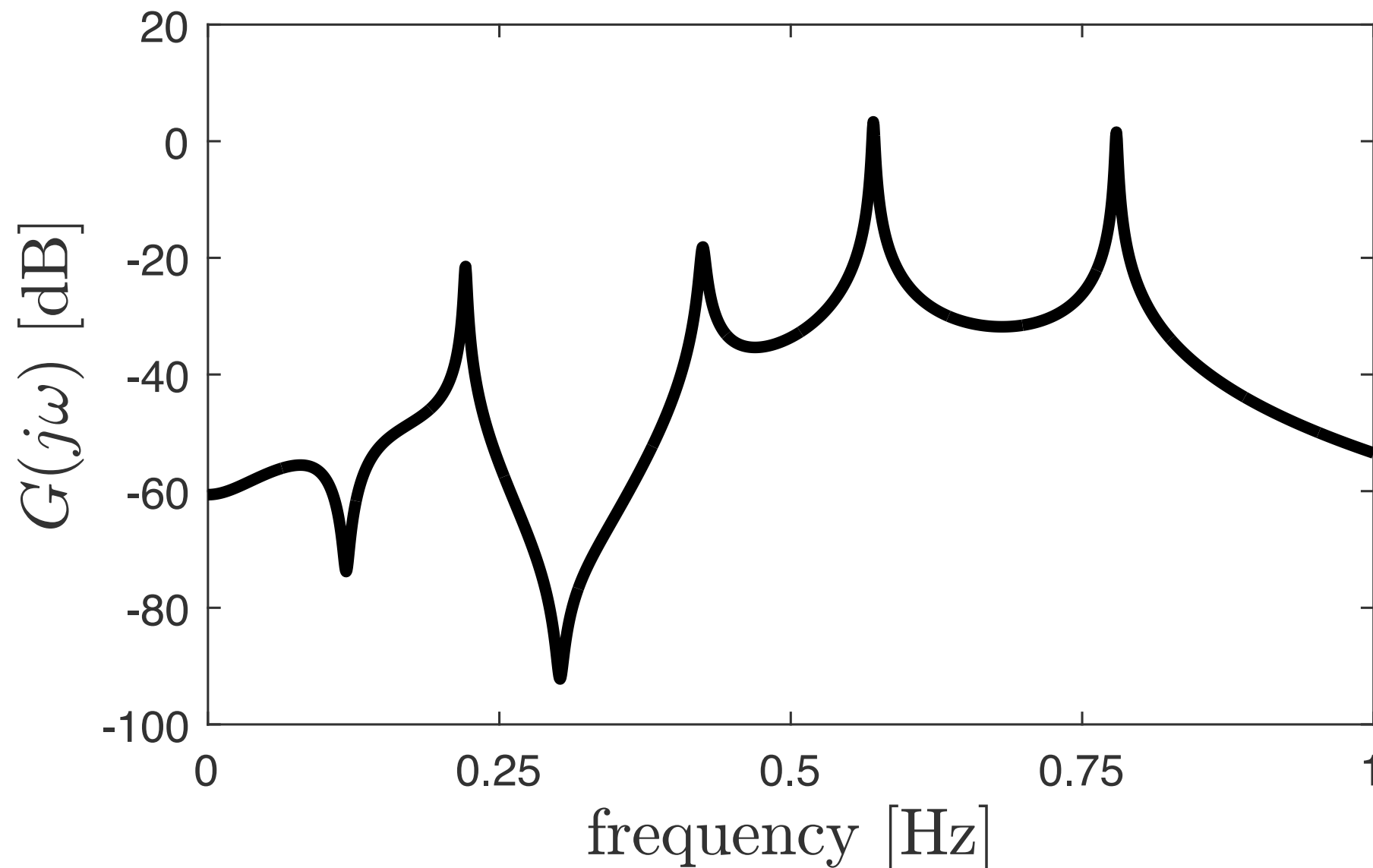
decay rate

Kernel

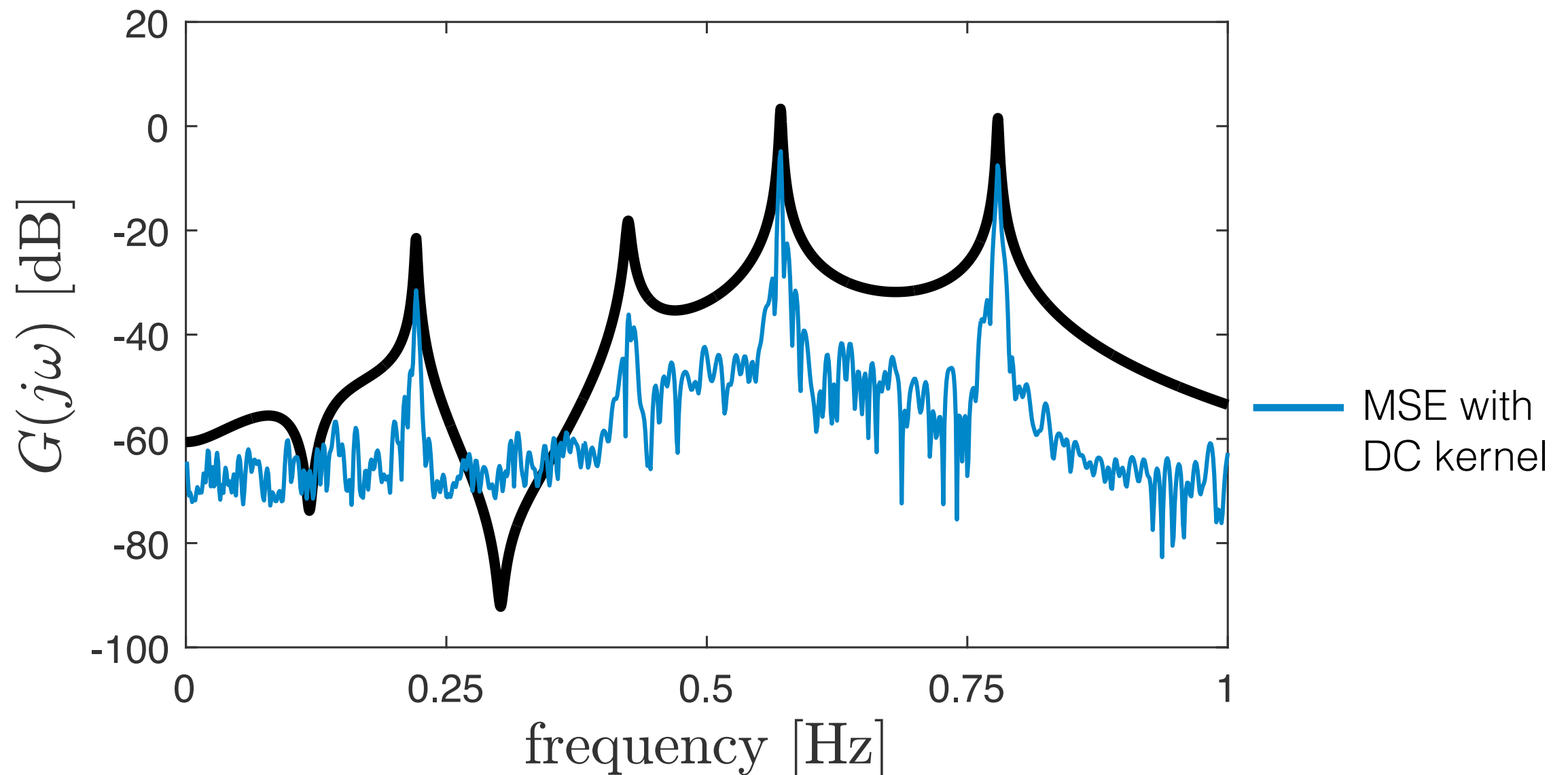


Challenge:

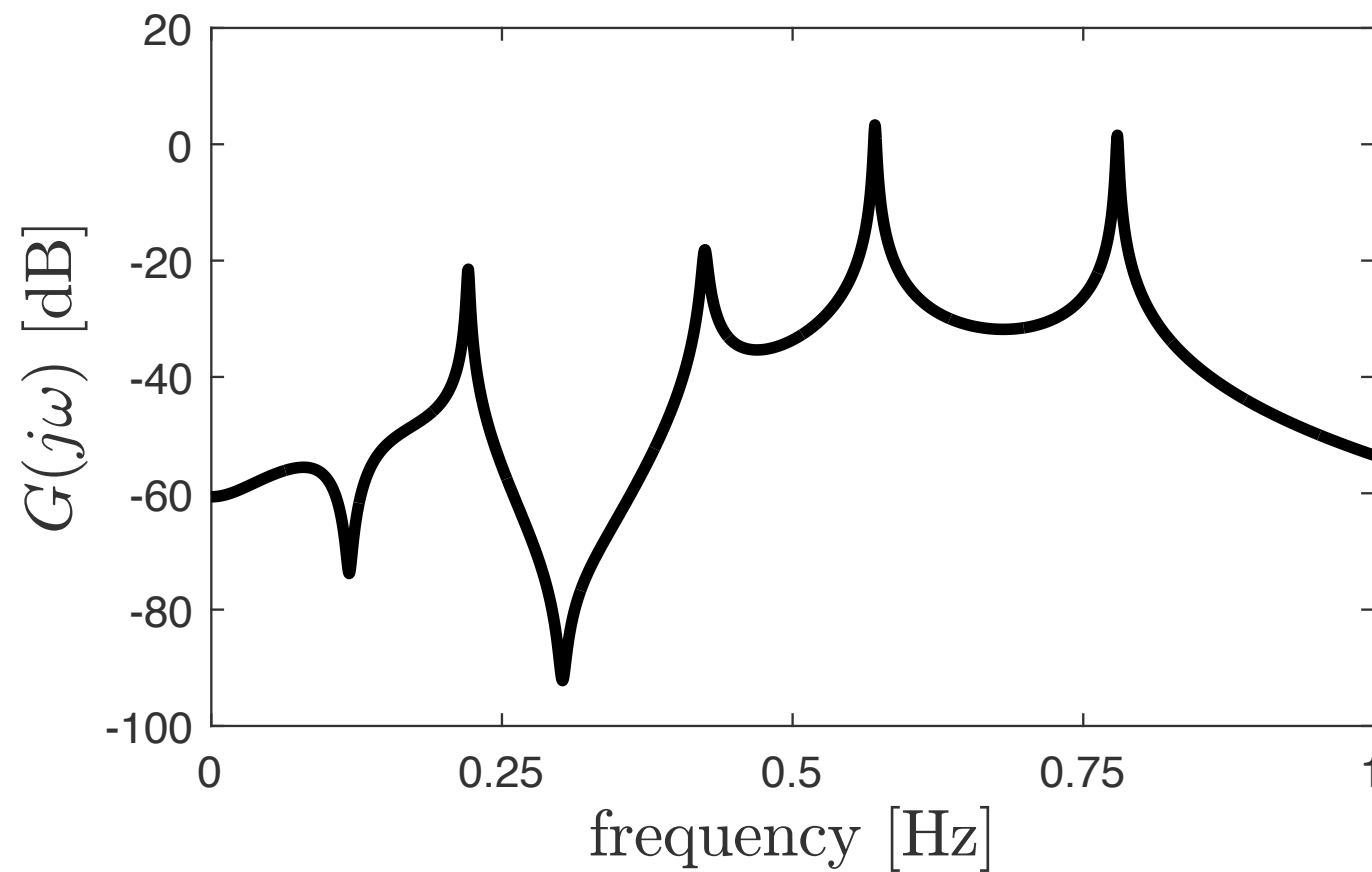
Lowly damped systems



Challenge: Lowly damped systems



Challenge: Lowly damped systems



$$g(t) = \sum_{n=1}^{n_p} g_n(t)$$

$$g_n(t) = e^{-\alpha_n t} (\zeta_{1,n} \cos(\omega_n t) + \zeta_{2,n} \sin(\omega_n t)) \quad p_n = -\alpha_n \pm j\omega_n$$

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$$p_n = -\alpha_n \pm j\omega_n$$

Diagram annotations:

- Red arrows point from "resonance frequency" to ω_n in both $\cos(\omega_n t)$ and $\sin(\omega_n t)$.
- A red arrow points from "(slow) decay rate" to α_n in $e^{-\alpha_n t}$.
- Green arrows point from "pole residue" to $\zeta_{1,n}$ and $\zeta_{2,n}$.

'Resonating' Kernel

$$k_{\text{res},n}(t, \tau) = \mathbb{E}\{g_n(t)g_n(\tau)\}$$

$$= e^{-\alpha_n(t+\tau)} (\gamma_{1,n} \cos(\omega_n(t - \tau)) + \gamma_{2,n} \cos(\omega_n(t + \tau)))$$

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$$g(t) = \sum_{n=1}^{n_p} g_n(t)$$

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Diagram annotations:

- Red arrow from α_n to (slow) decay rate
- Red arrow from ω_n to resonance frequency
- Green arrows from $\zeta_{1,n}$ and $\zeta_{2,n}$ to pole residue

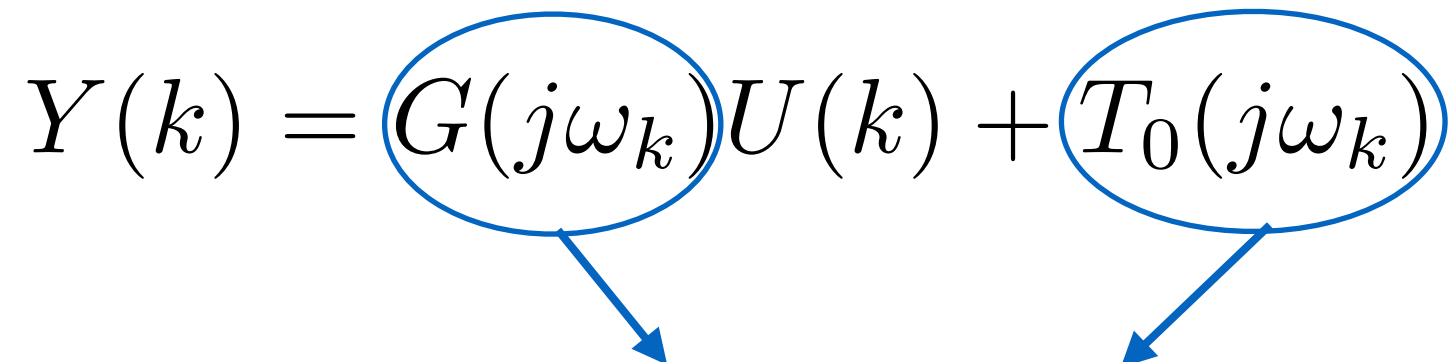
'Resonating' Kernel

Starting values hyper parameters?

$$k_{\text{res},n}(t, \tau) = \mathbb{E}\{g_n(t)g_n(\tau)\}$$

$$= e^{-\alpha_n(t+\tau)} (\gamma_{1,n} \cos(\omega_n(t - \tau)) + \gamma_{2,n} \cos(\omega_n(t + \tau)))$$

Local Rational Method

$$Y(k) = G(j\omega_k)U(k) + T_0(j\omega_k)$$


locally approximated by rational forms

$$G(j\Omega_{k+r}) \approx \frac{B_k^c(j\Omega_r)}{A_k^c(j\Omega_r)}$$

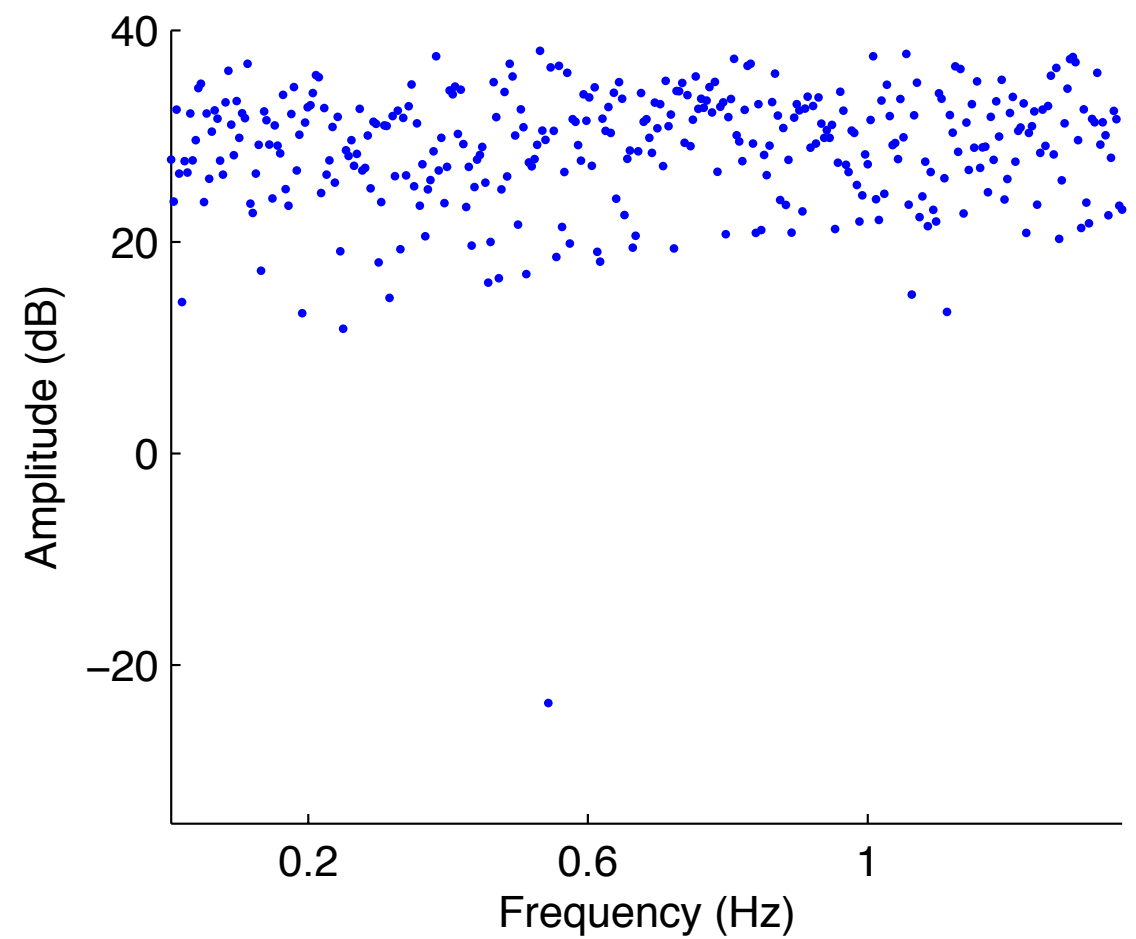
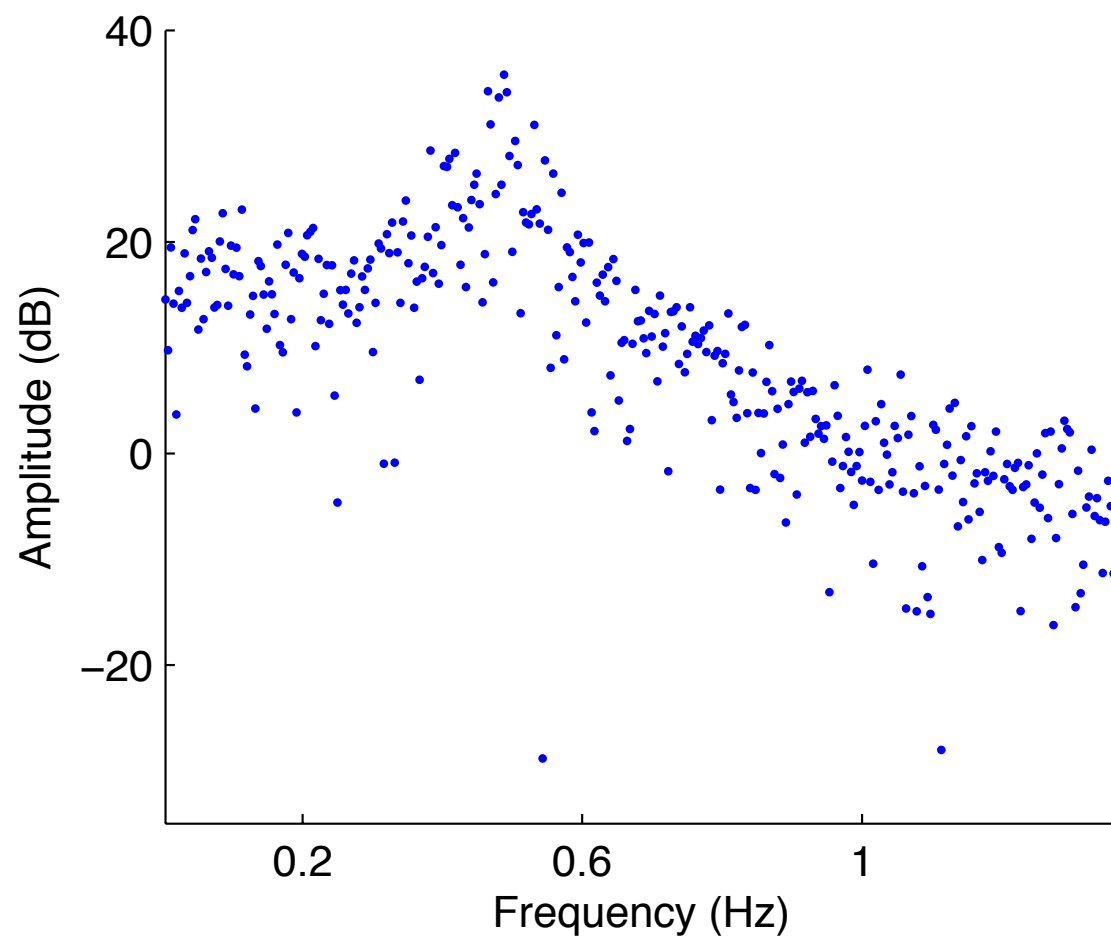
$$T_0(j\Omega_{k+r}) \approx \frac{I_k^c(j\Omega_r)}{A_k^c(j\Omega_r)}$$

for $r = -n, \dots, n$

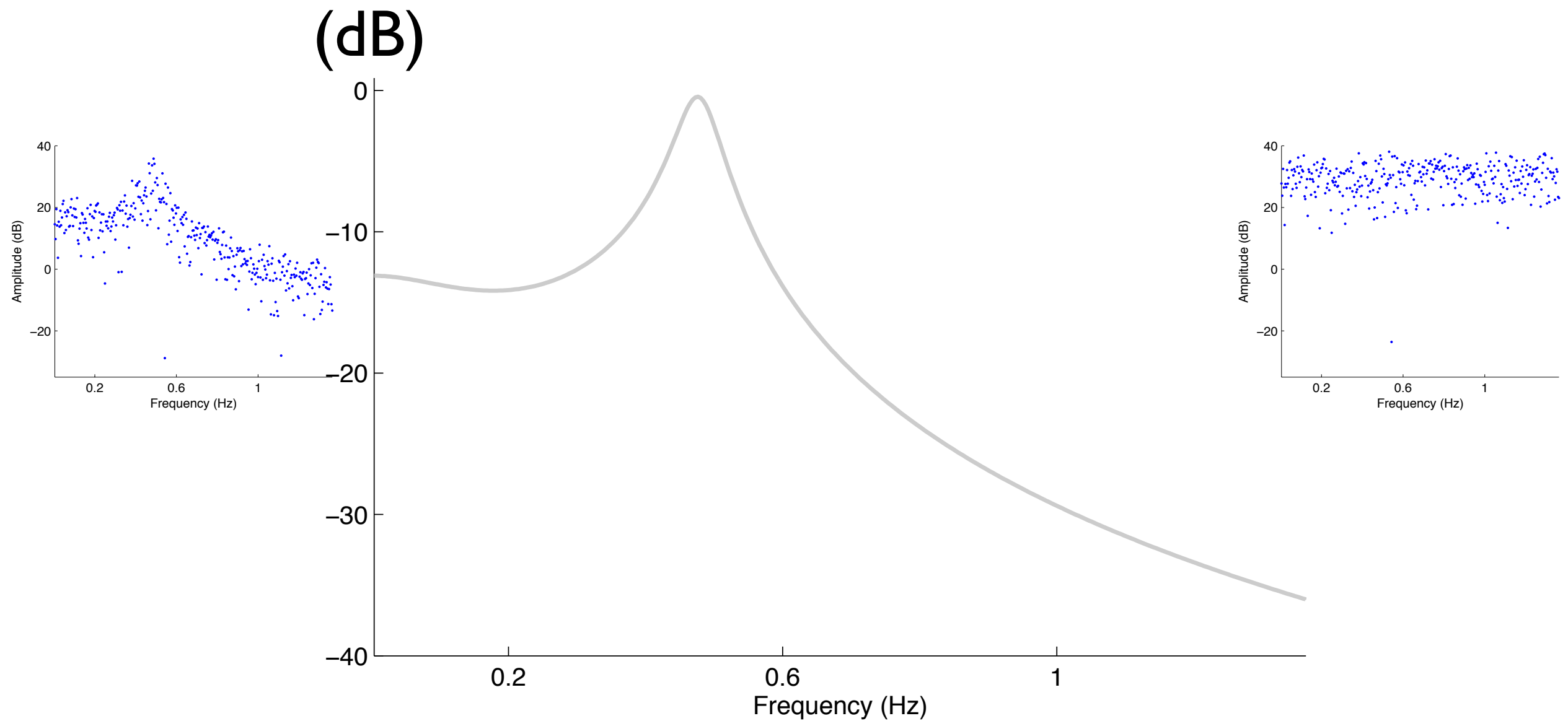
local frequency band around k



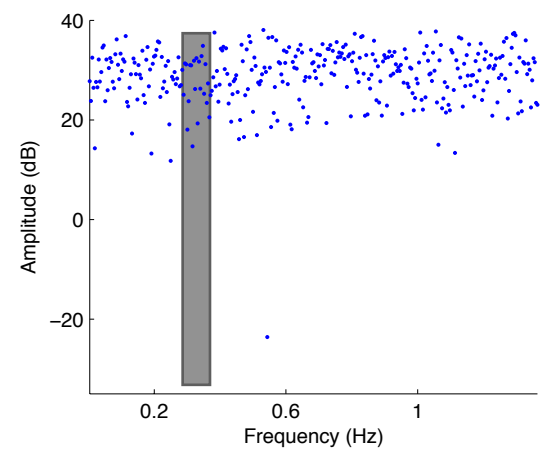
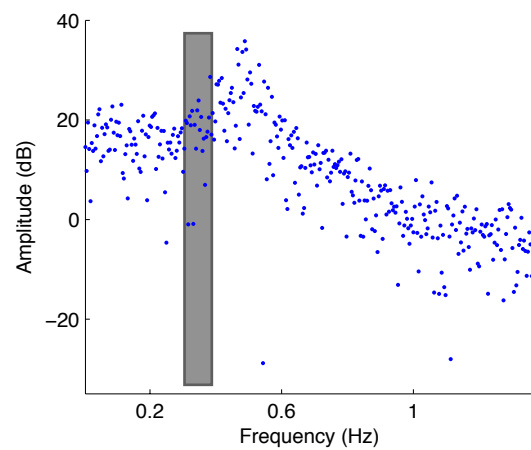
$$Y \approx \hat{G}U + \hat{T}_0$$



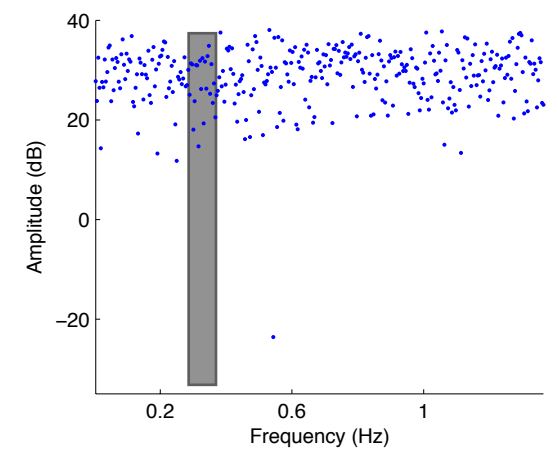
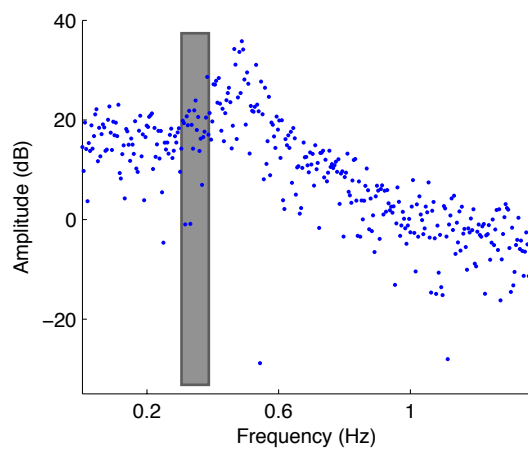
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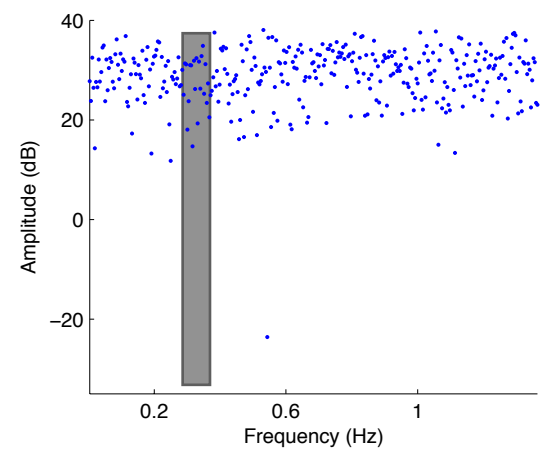
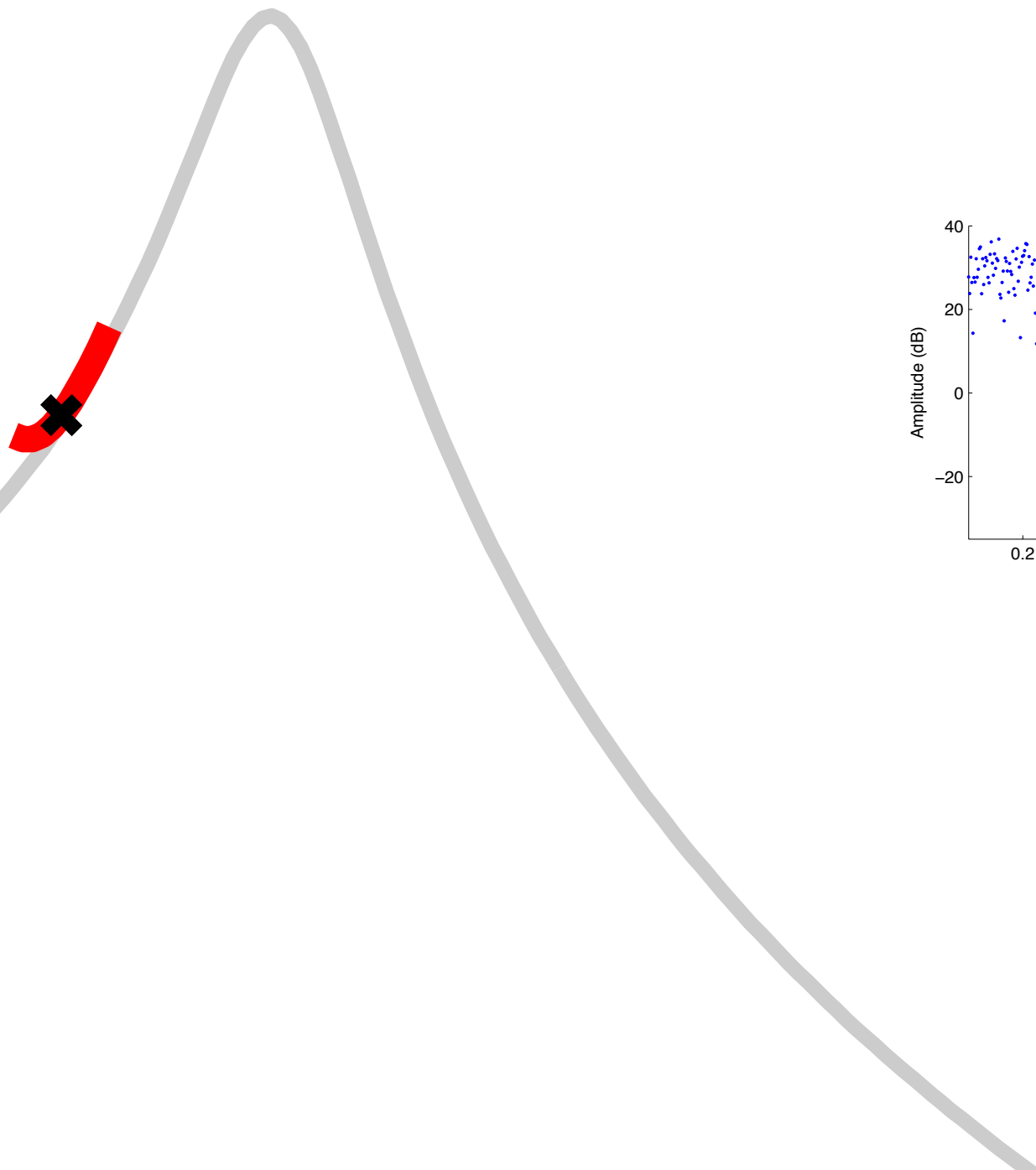
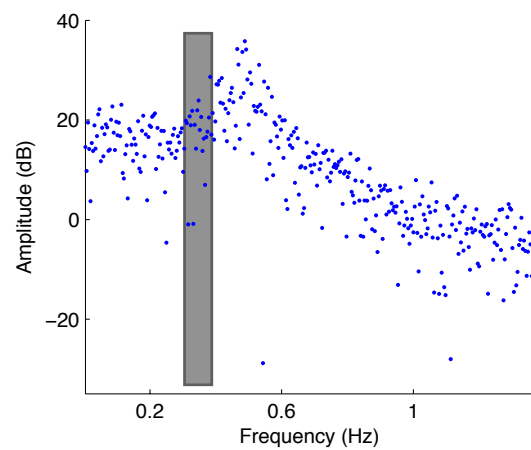
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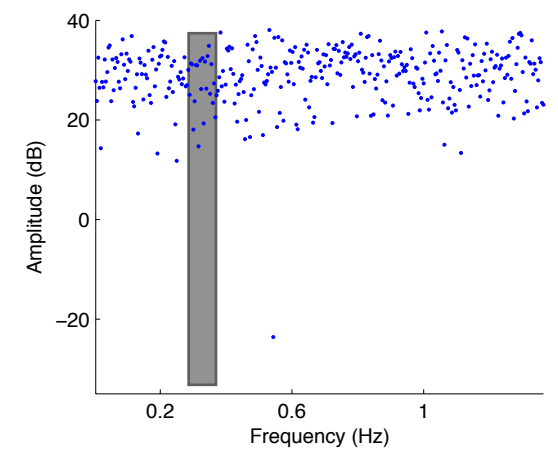
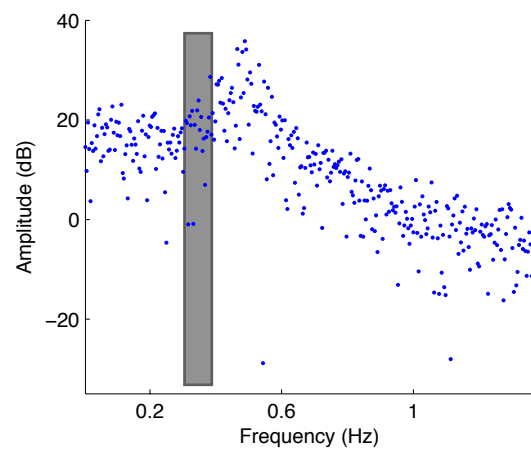
$$Y \approx \hat{G}U + \hat{T}_0$$



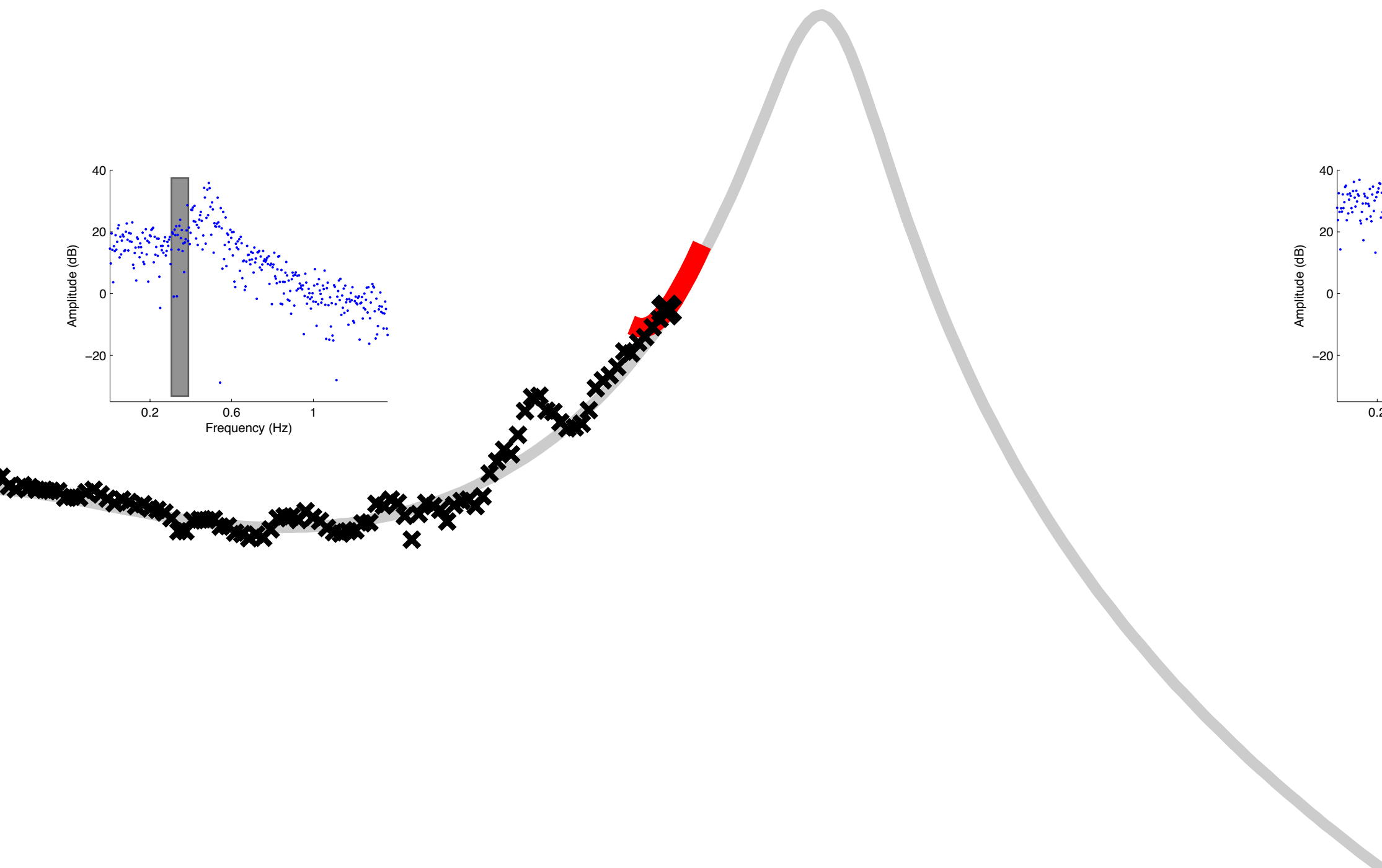
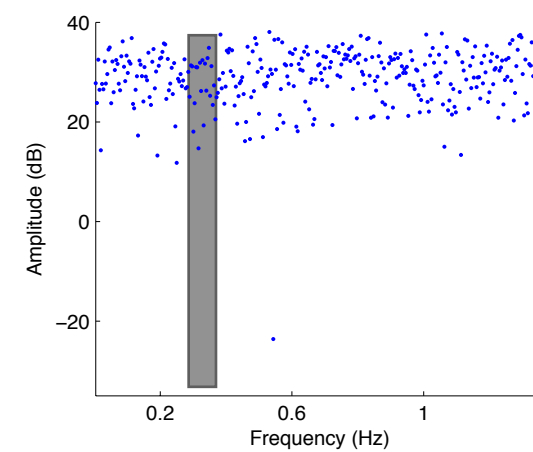
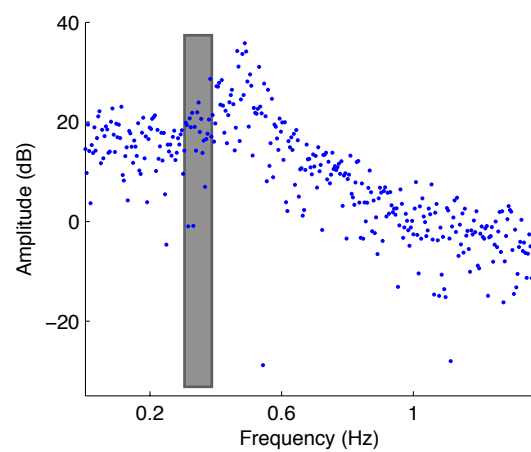
$$Y \approx \hat{G}U + \hat{T}_0$$



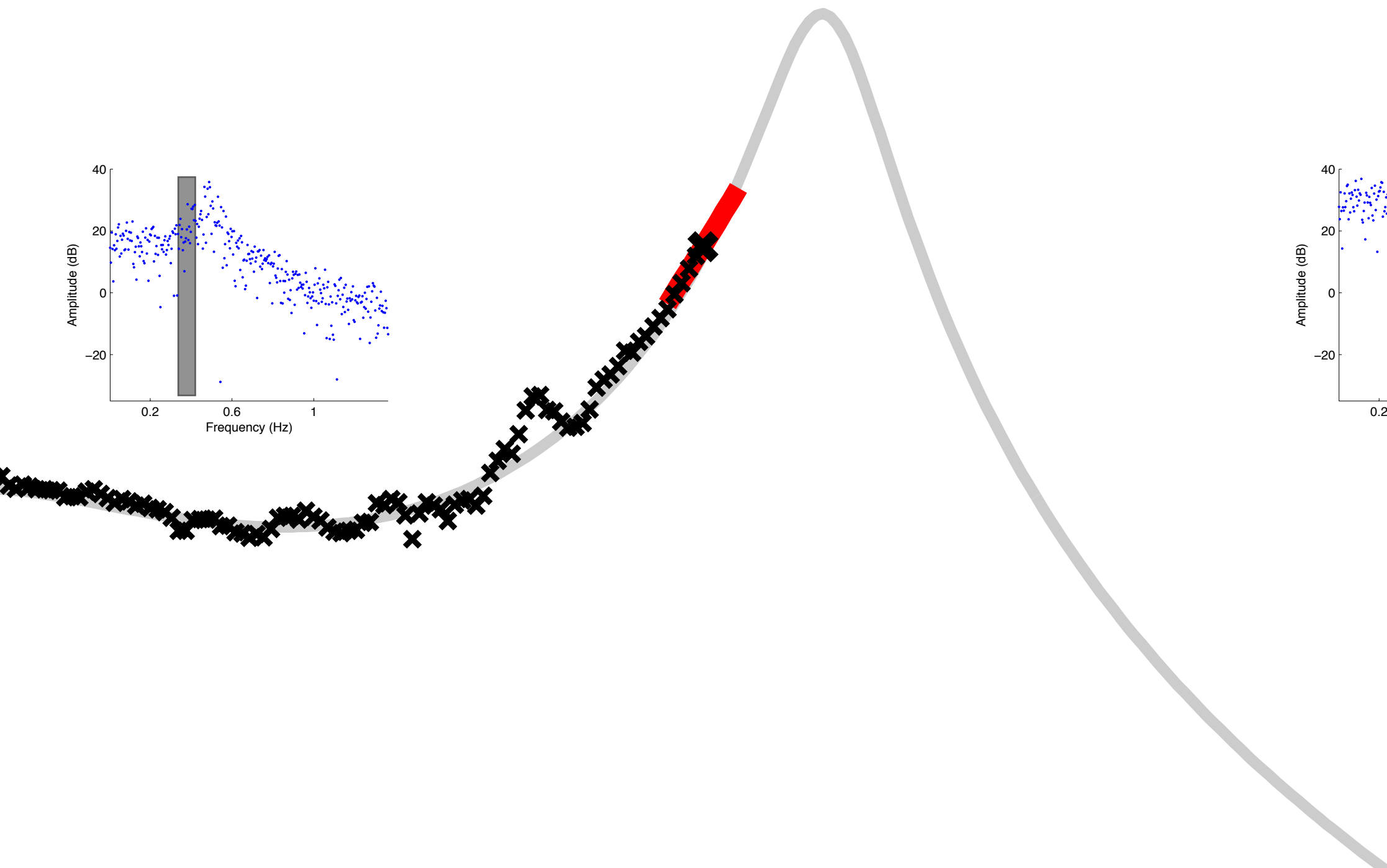
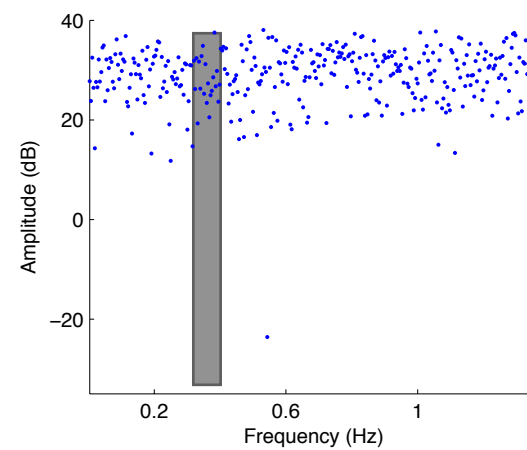
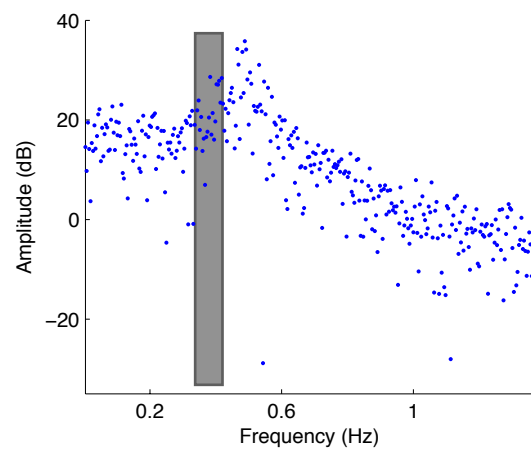
$$Y \approx \hat{G}U + \hat{T}_0$$



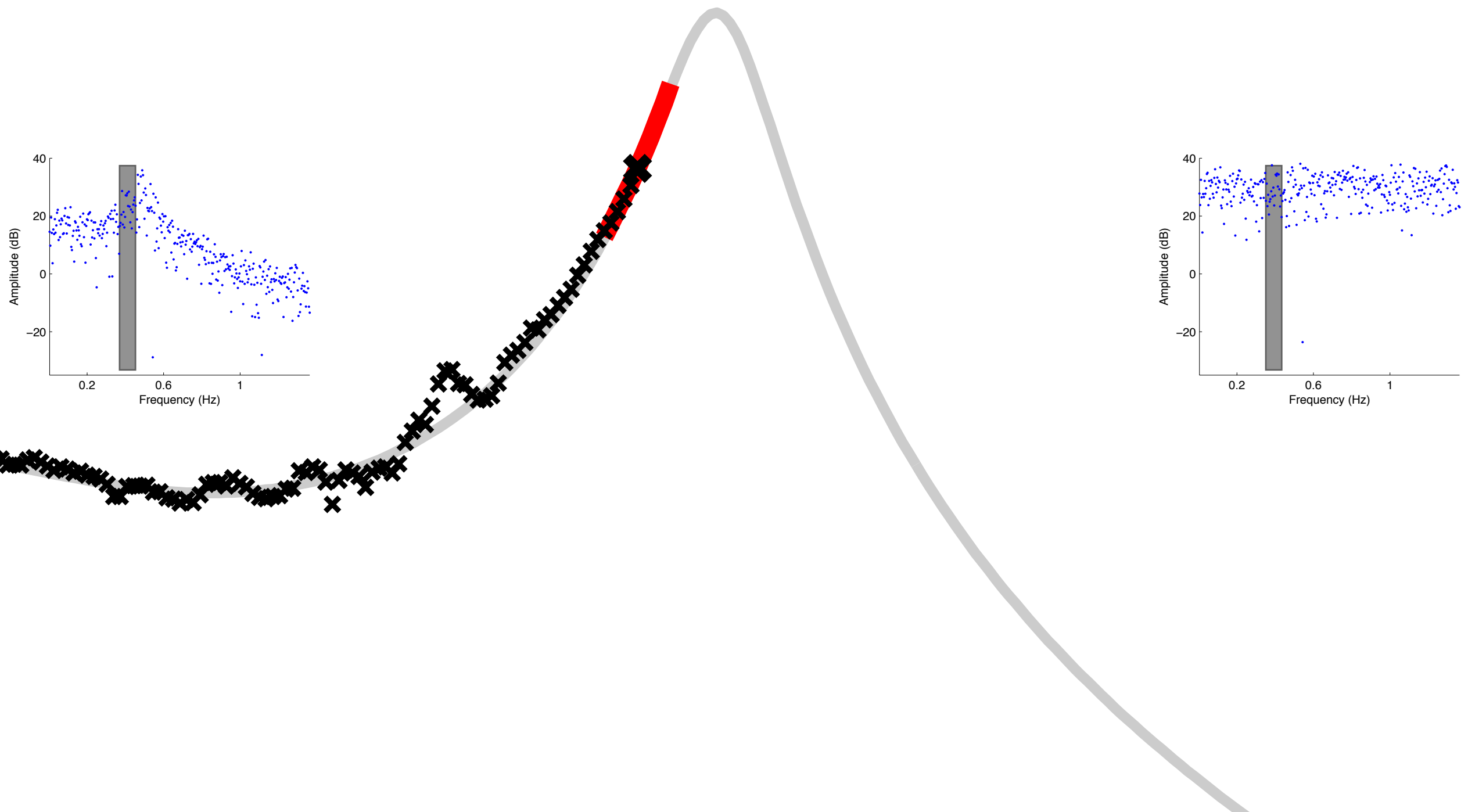
$$Y \approx \hat{G}U + \hat{T}_0$$



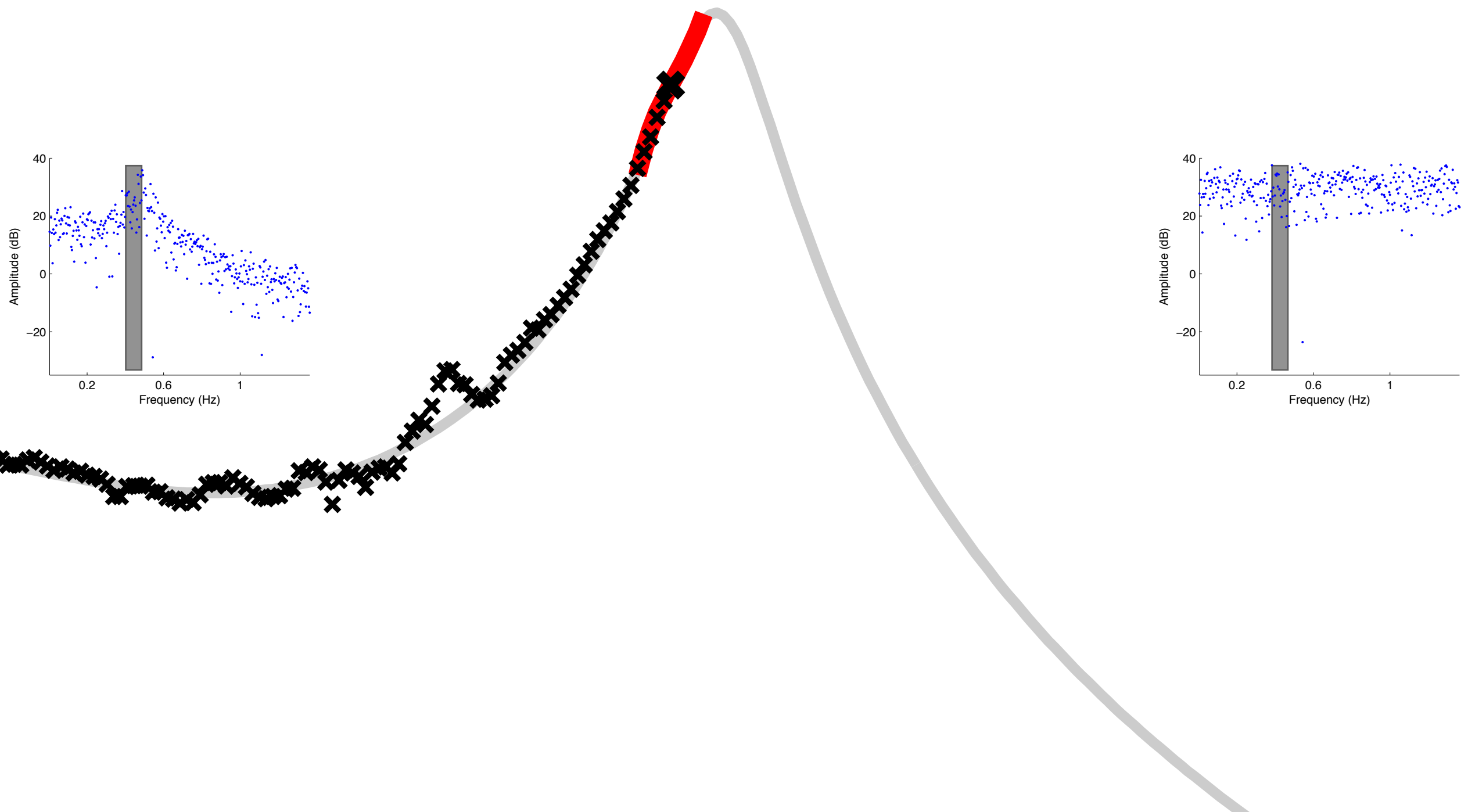
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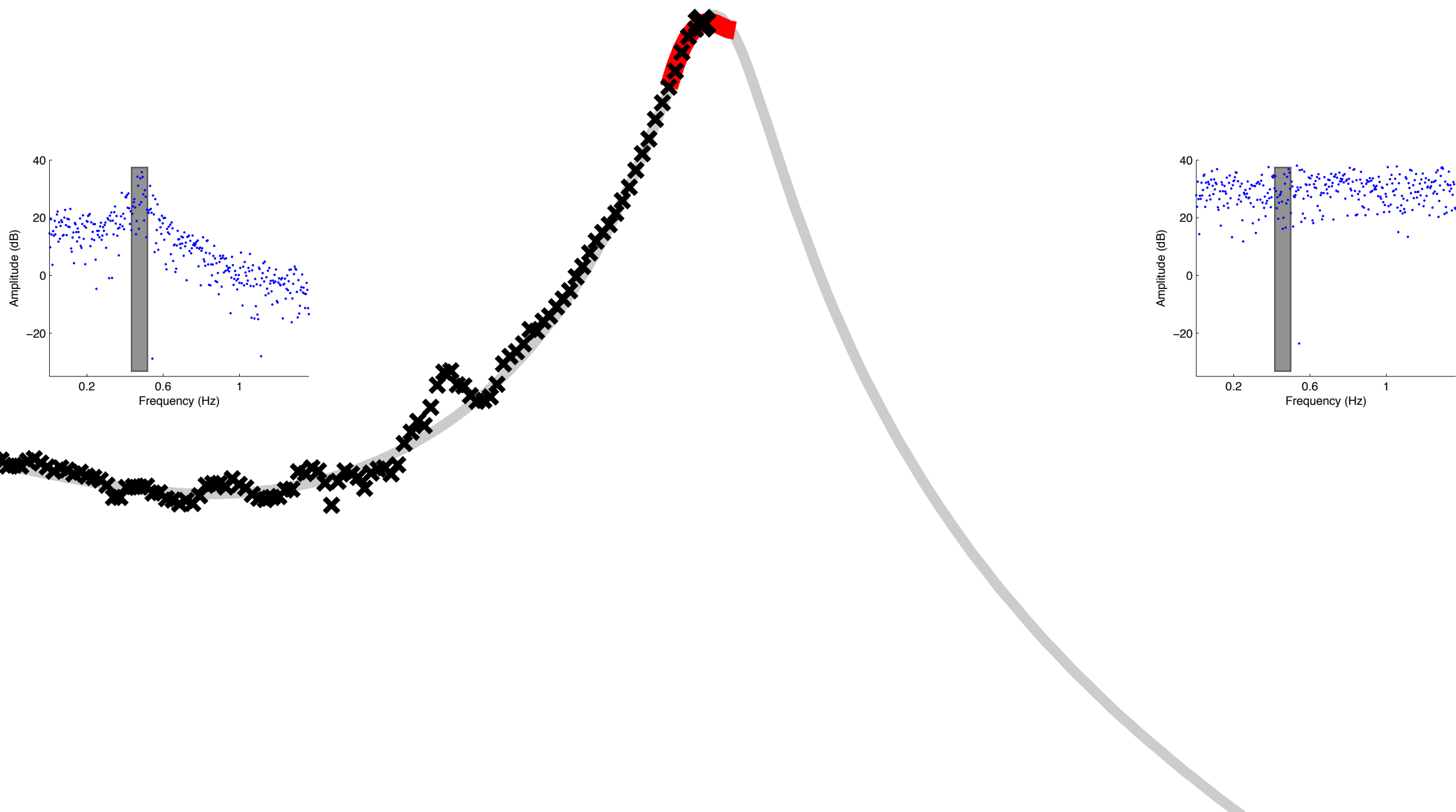
$$Y \approx \hat{G}U + \hat{T}_0$$



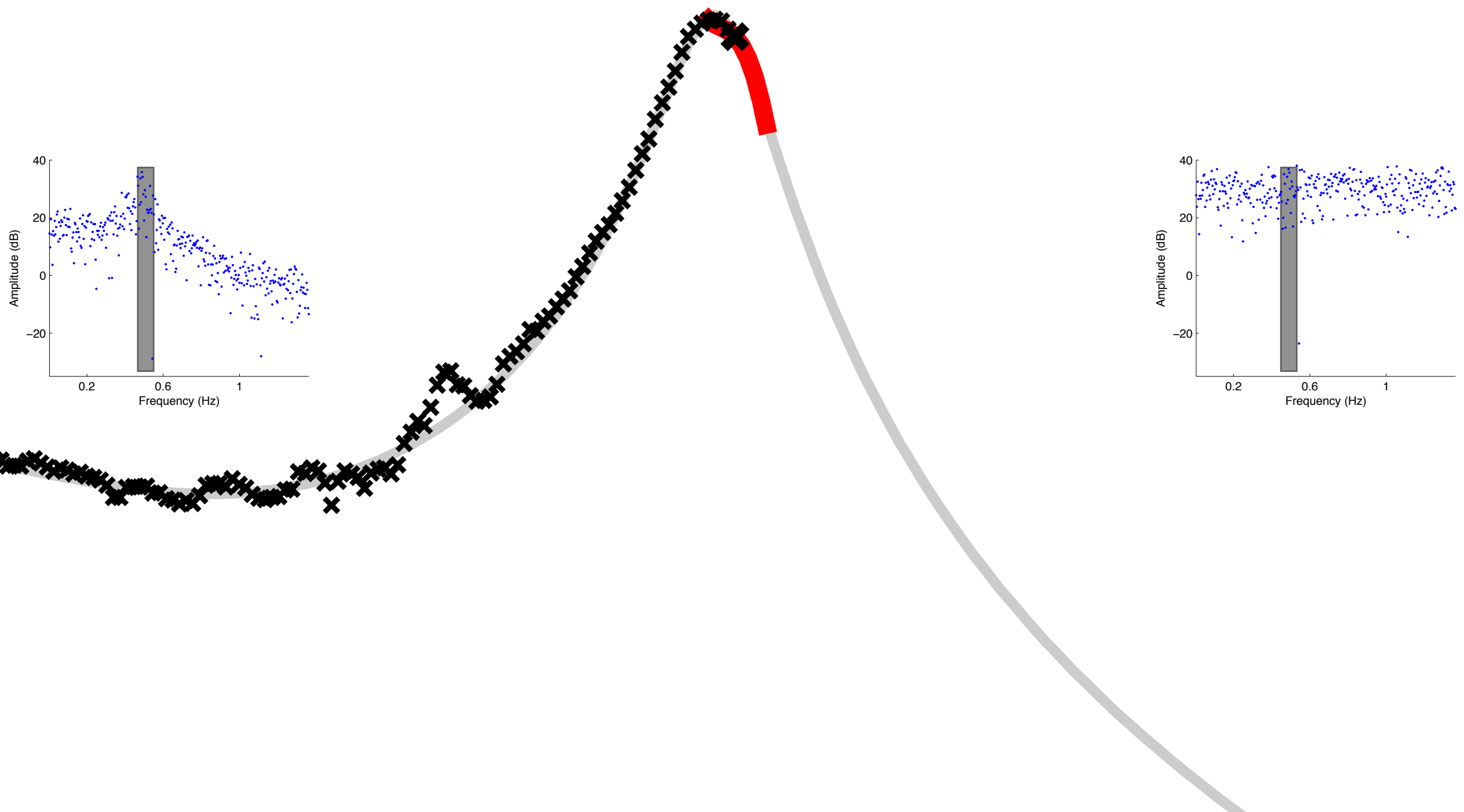
$$Y \approx \hat{G}U + \hat{T}_0$$



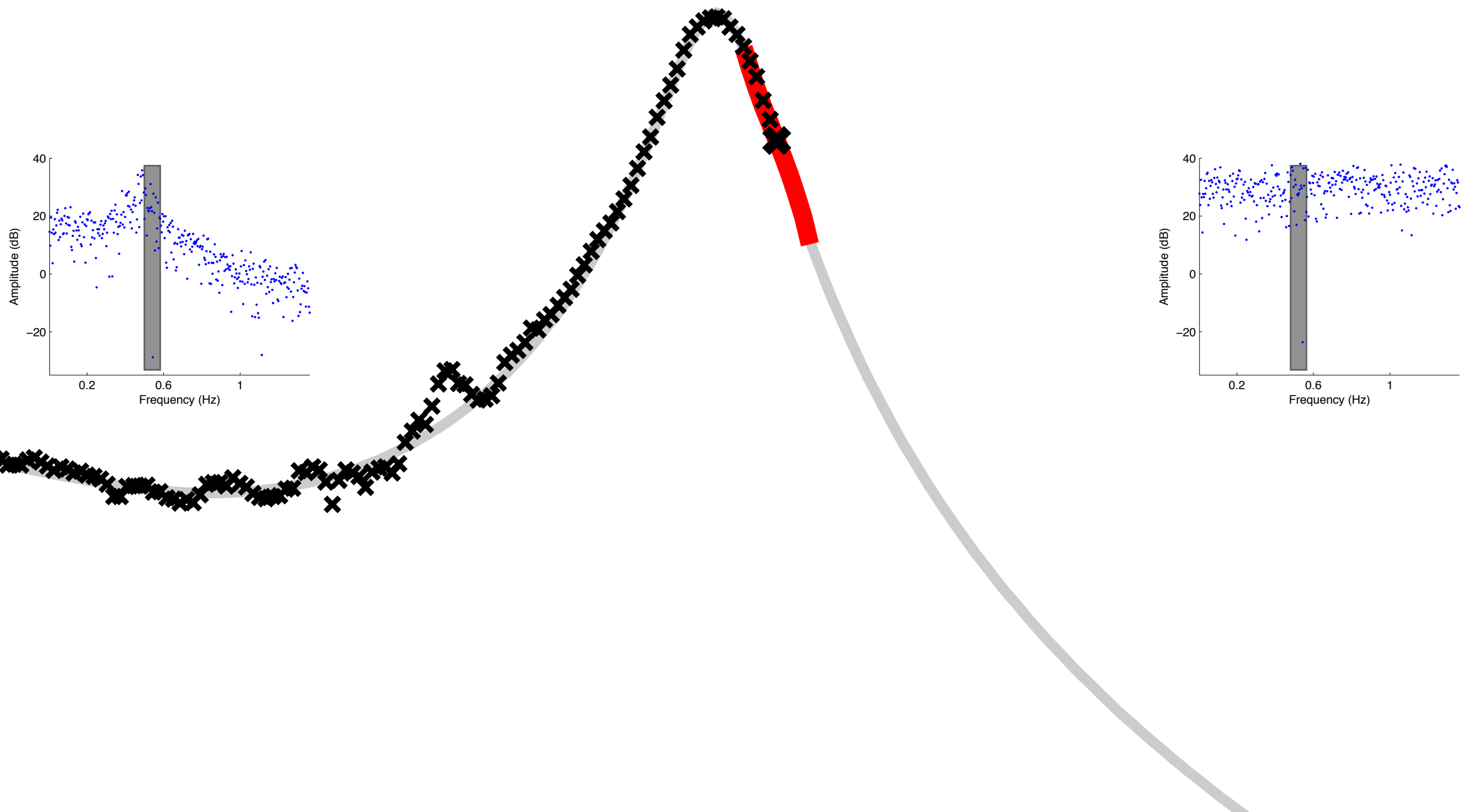
$$Y \approx \hat{G}U + \hat{T}_0$$



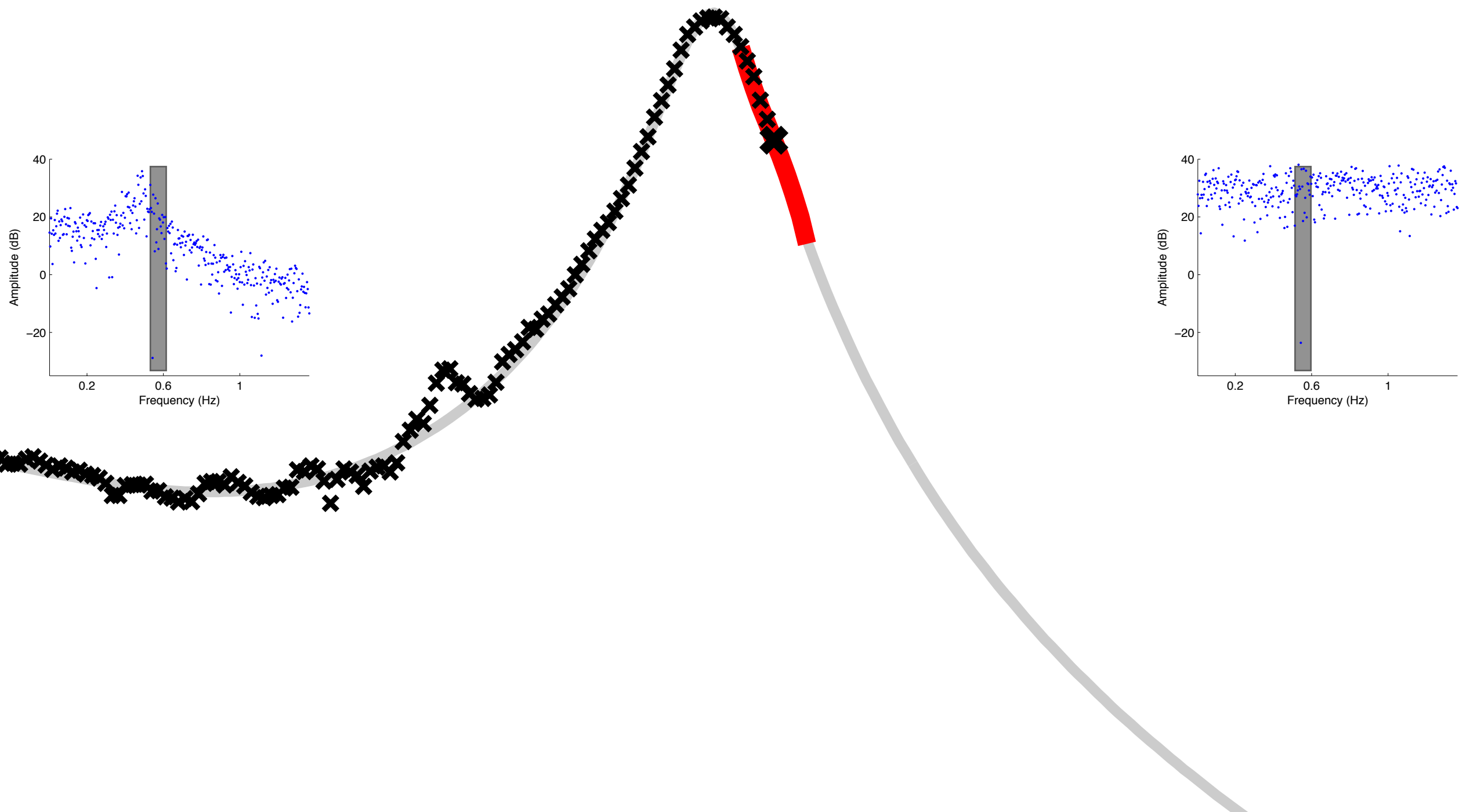
$$Y \approx \hat{G}U + \hat{T}_0$$



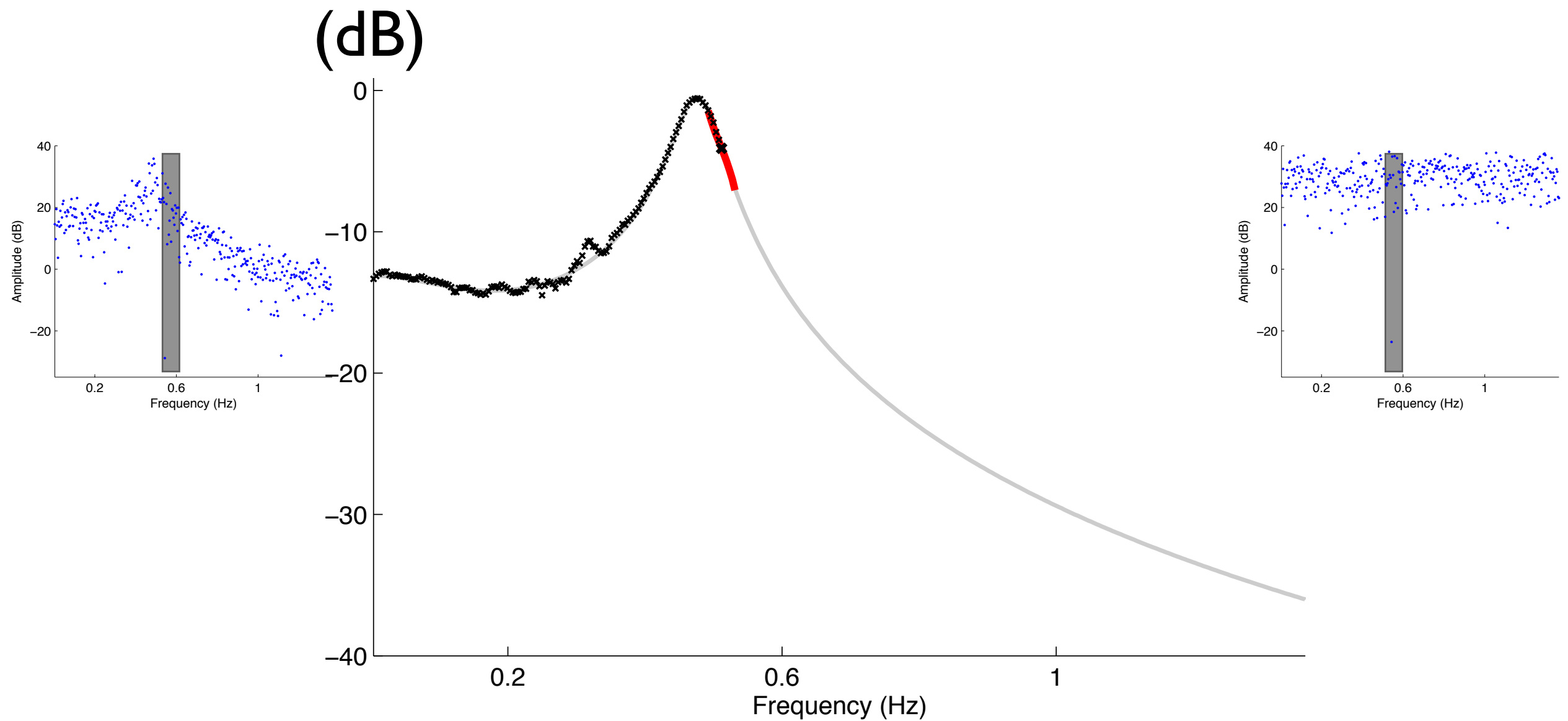
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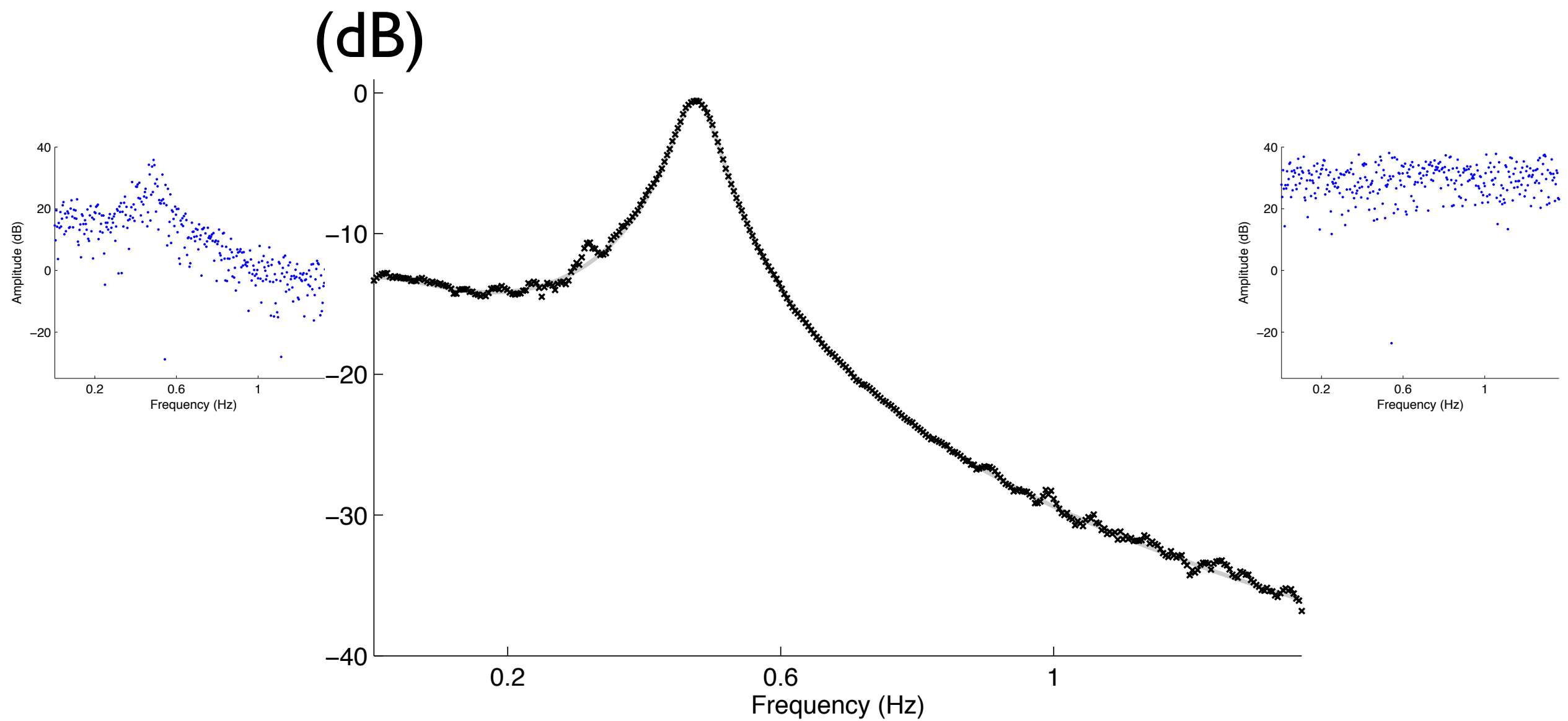
$$Y \approx \hat{G}U + \hat{T}_0$$



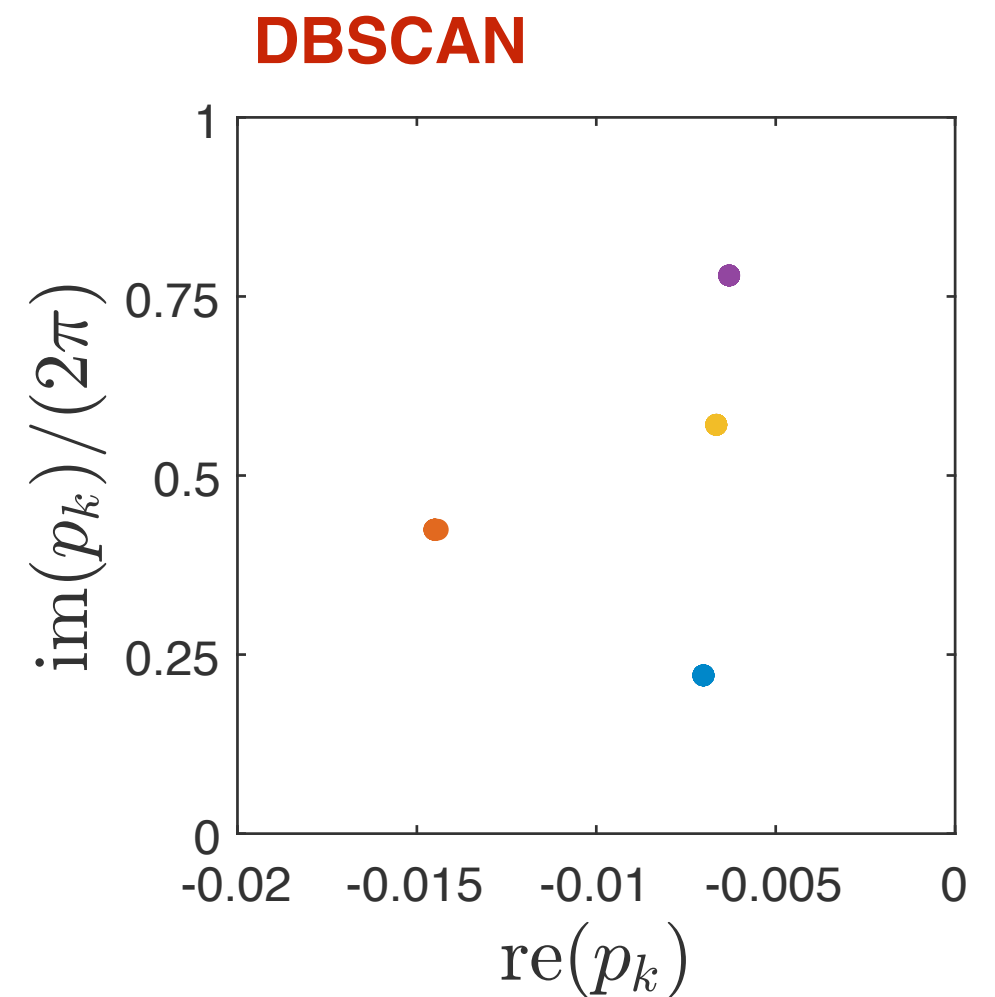
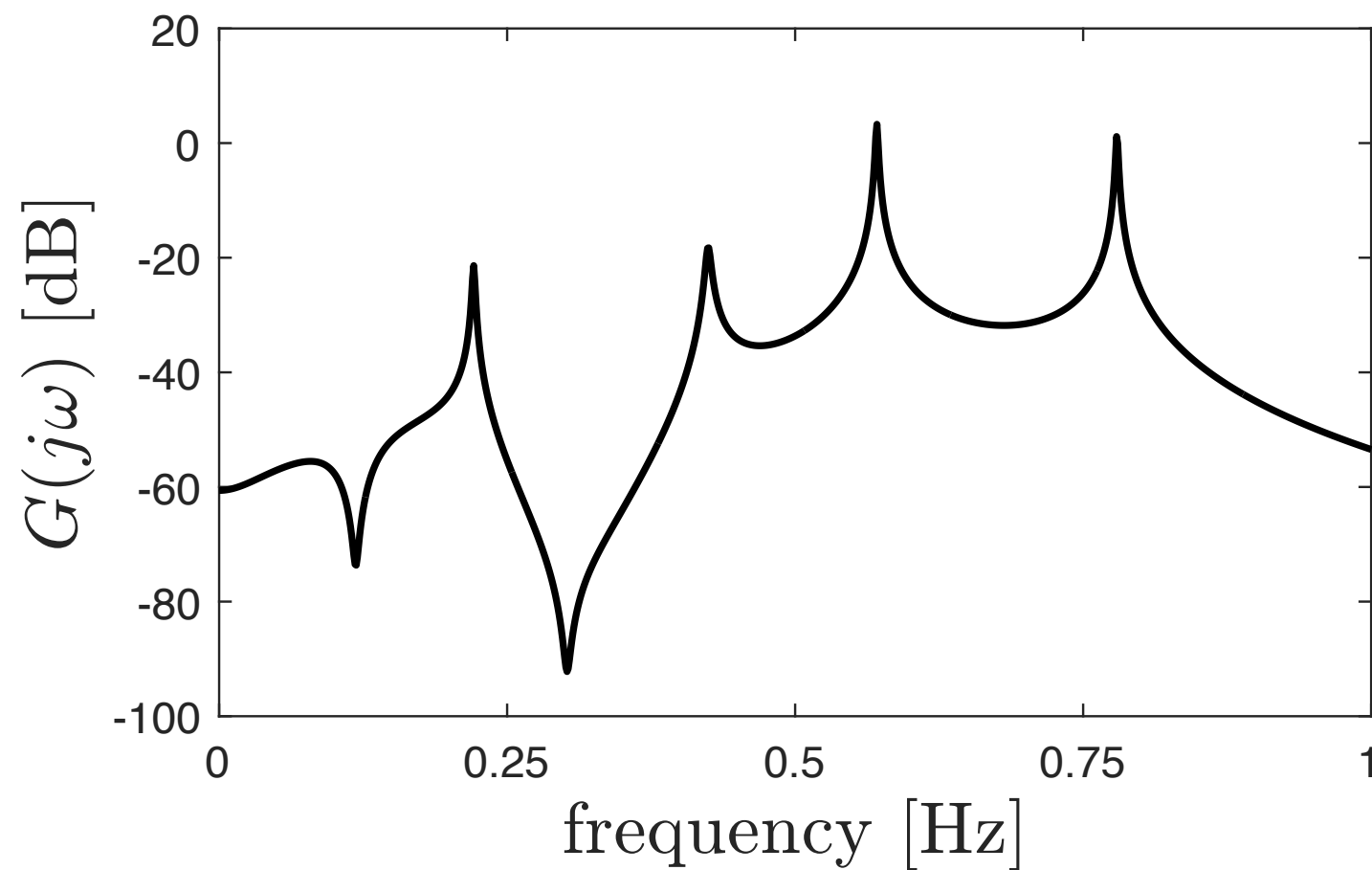
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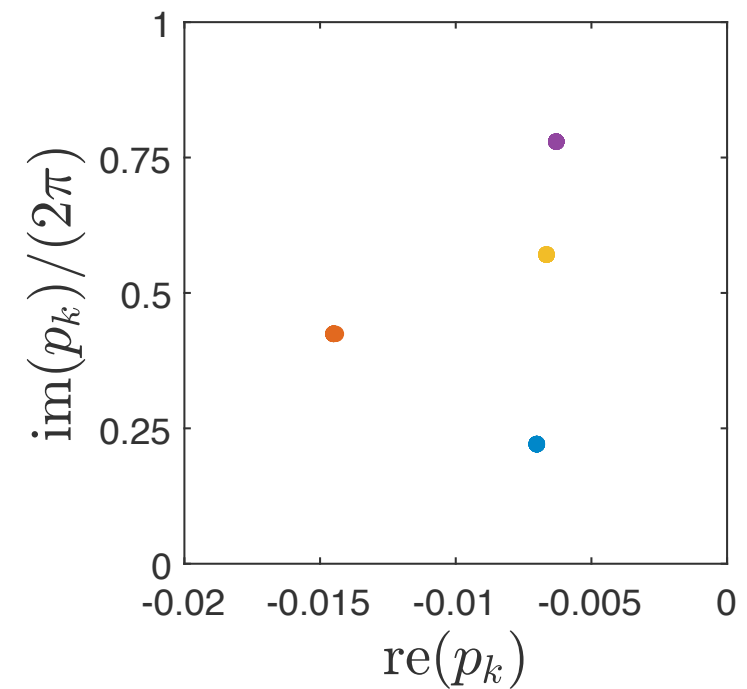
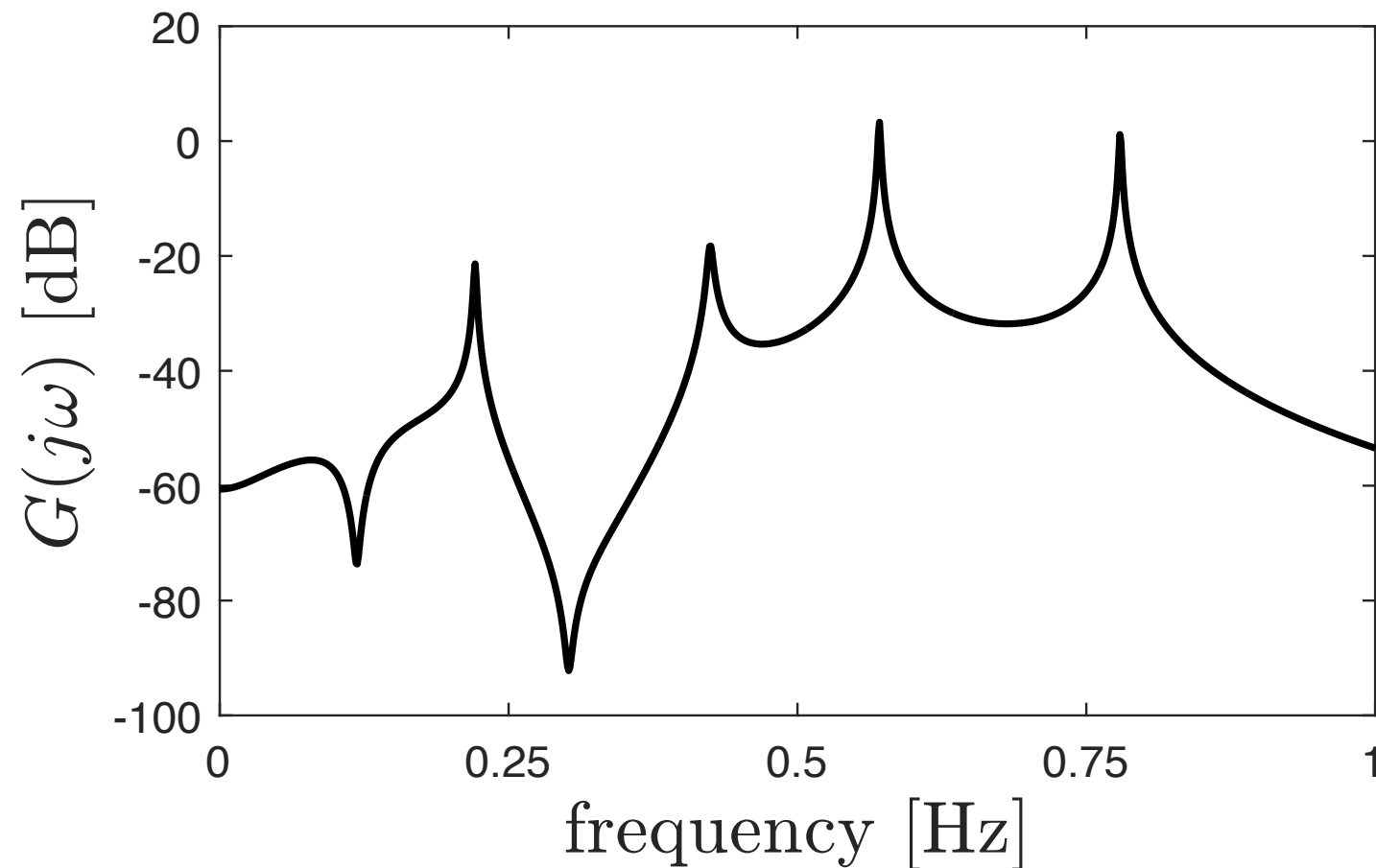
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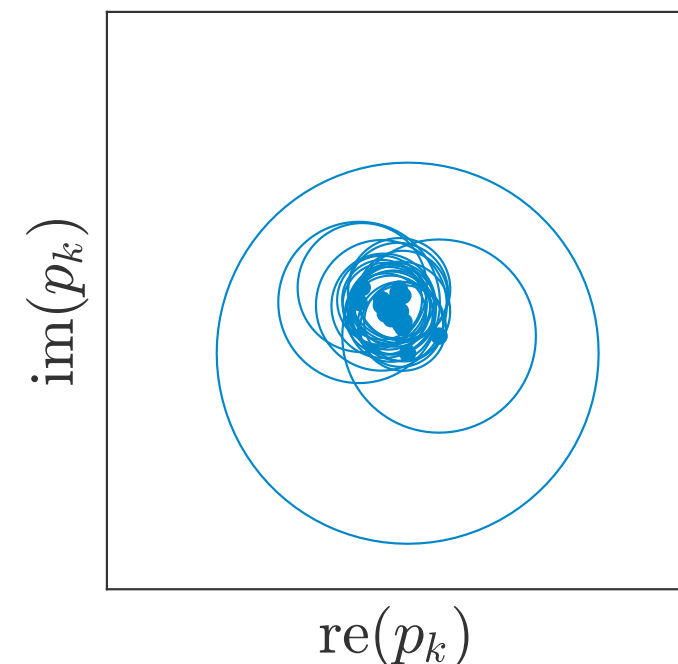
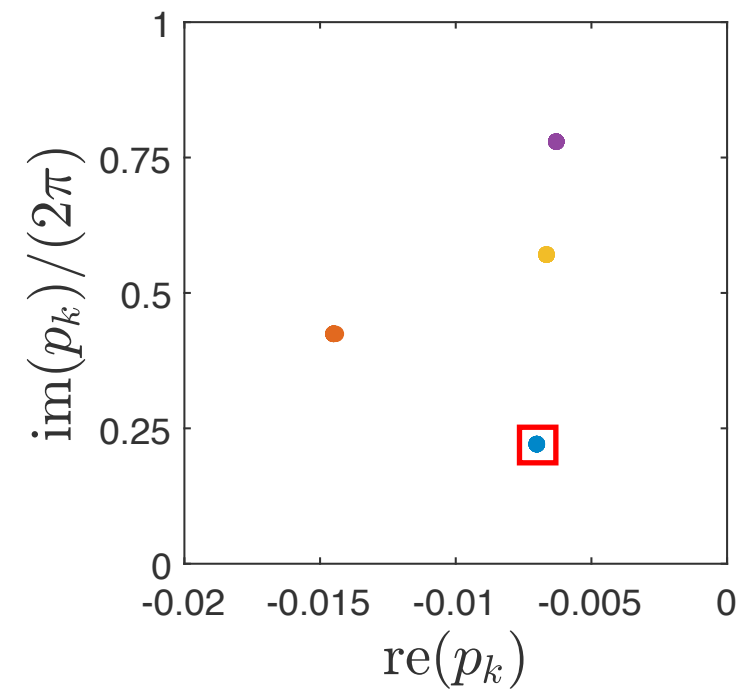
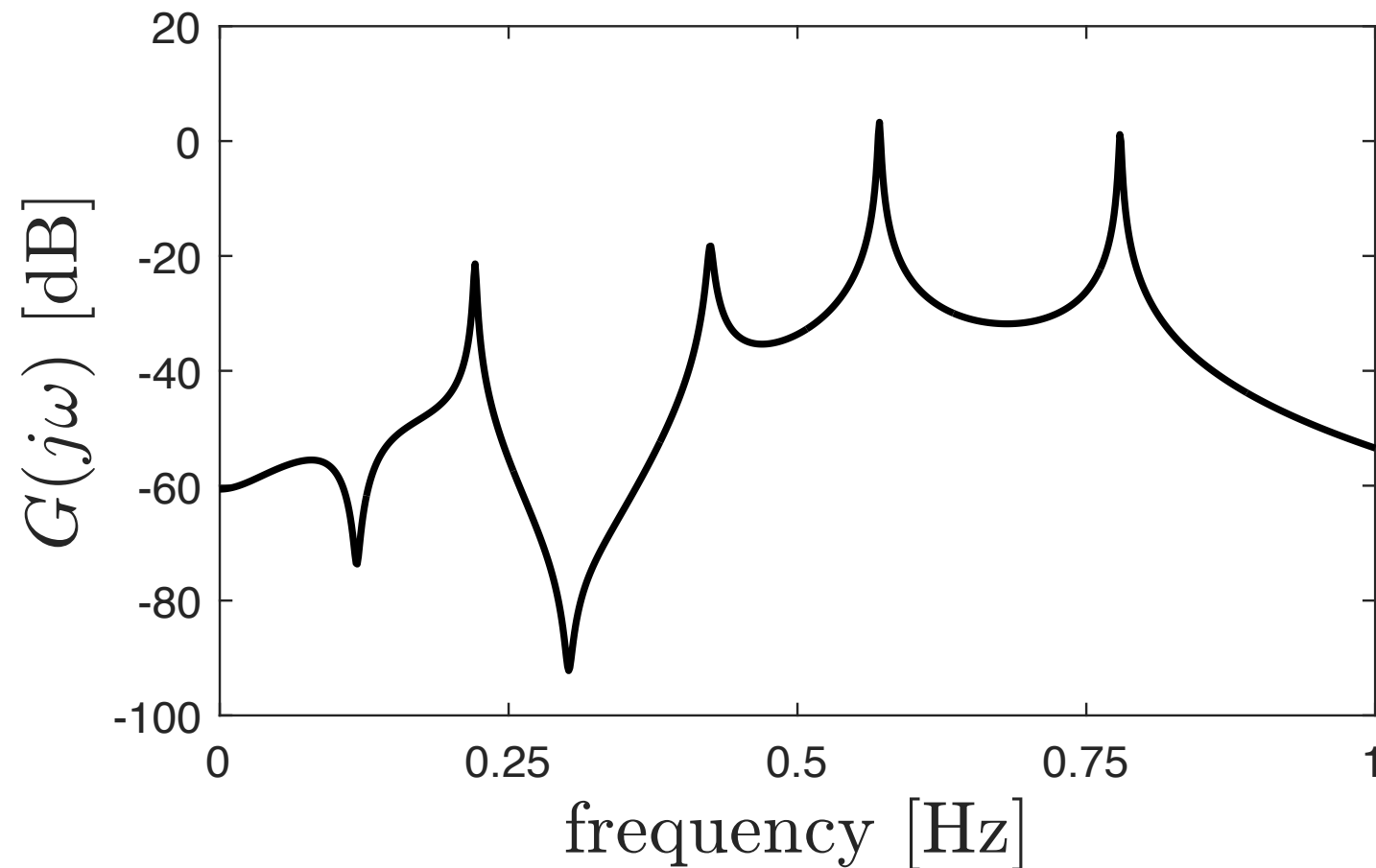
LRM $\rightarrow$ Estimate Poles



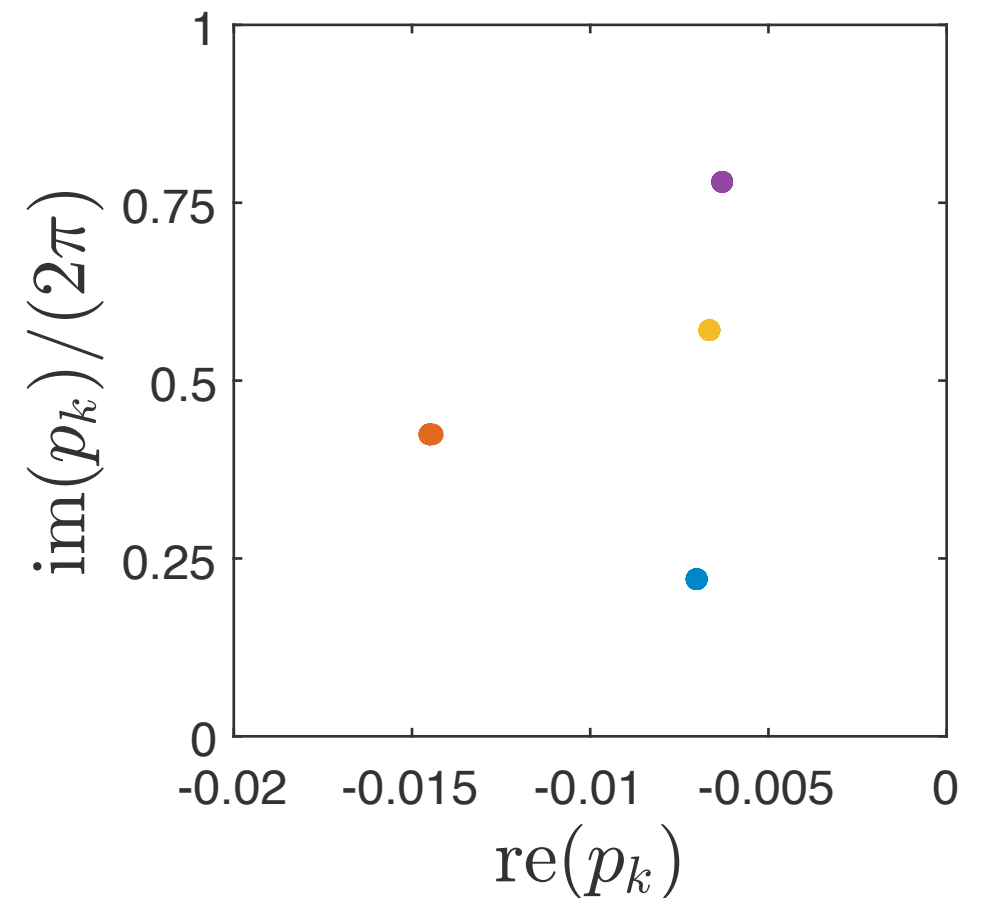
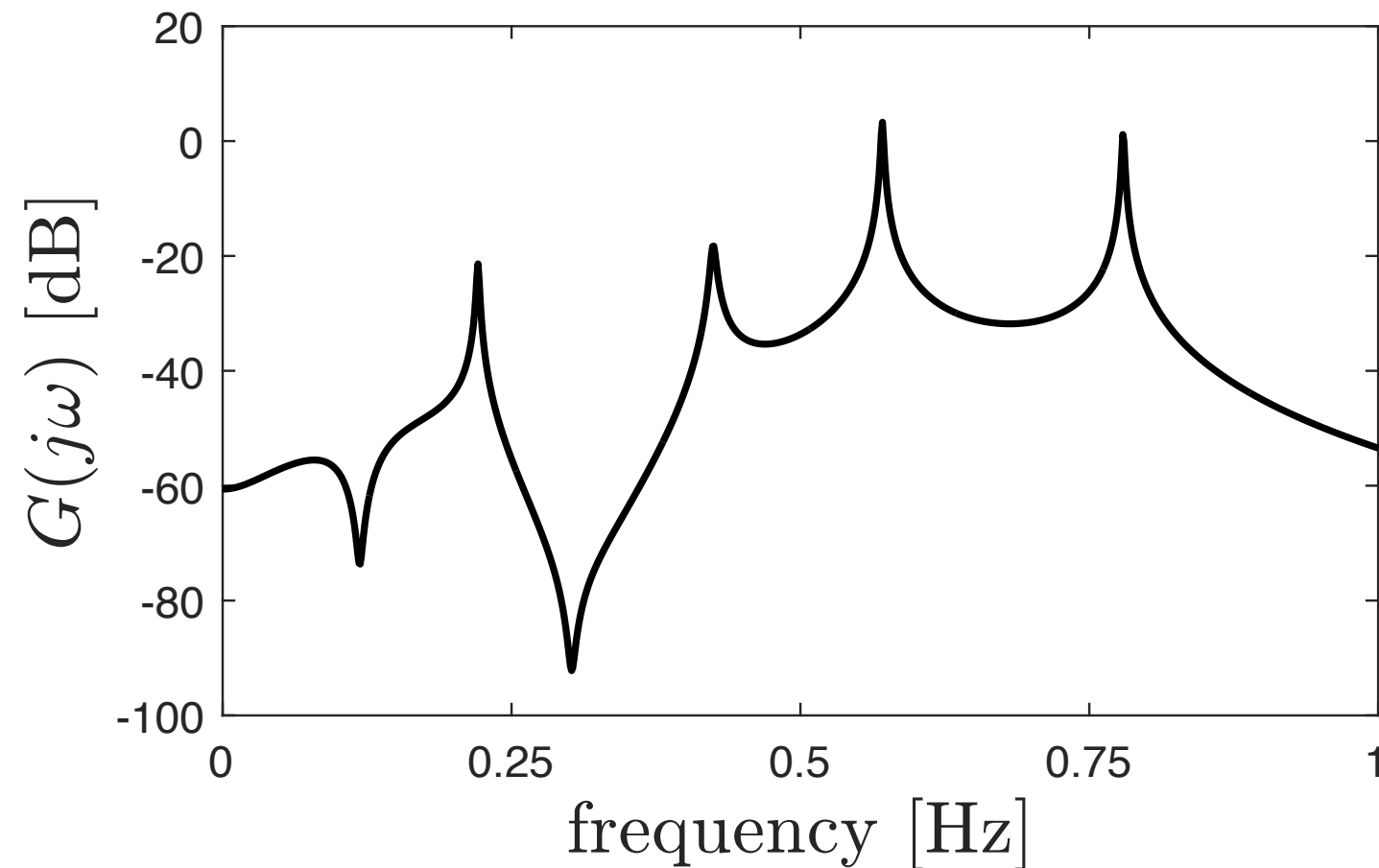
LRM $\rightarrow$ Estimate Poles



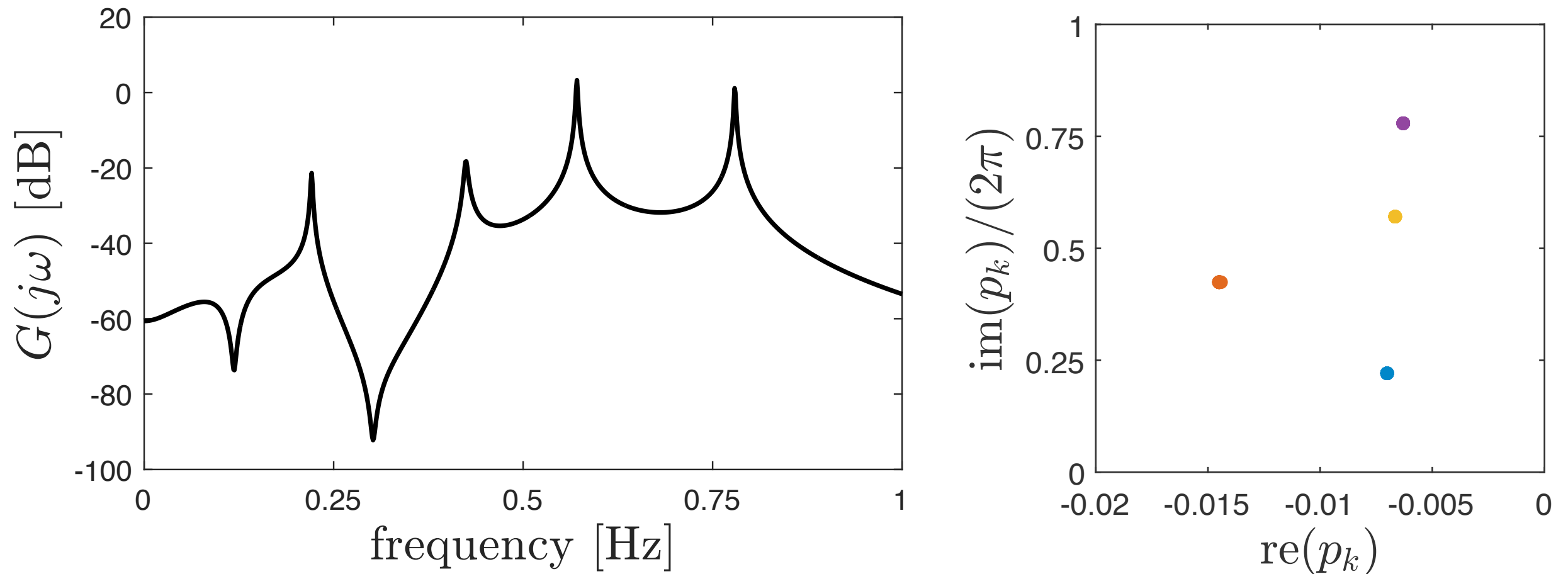
LRM $\rightarrow$ Estimate Poles



LRM $\rightarrow$ Estimate Poles



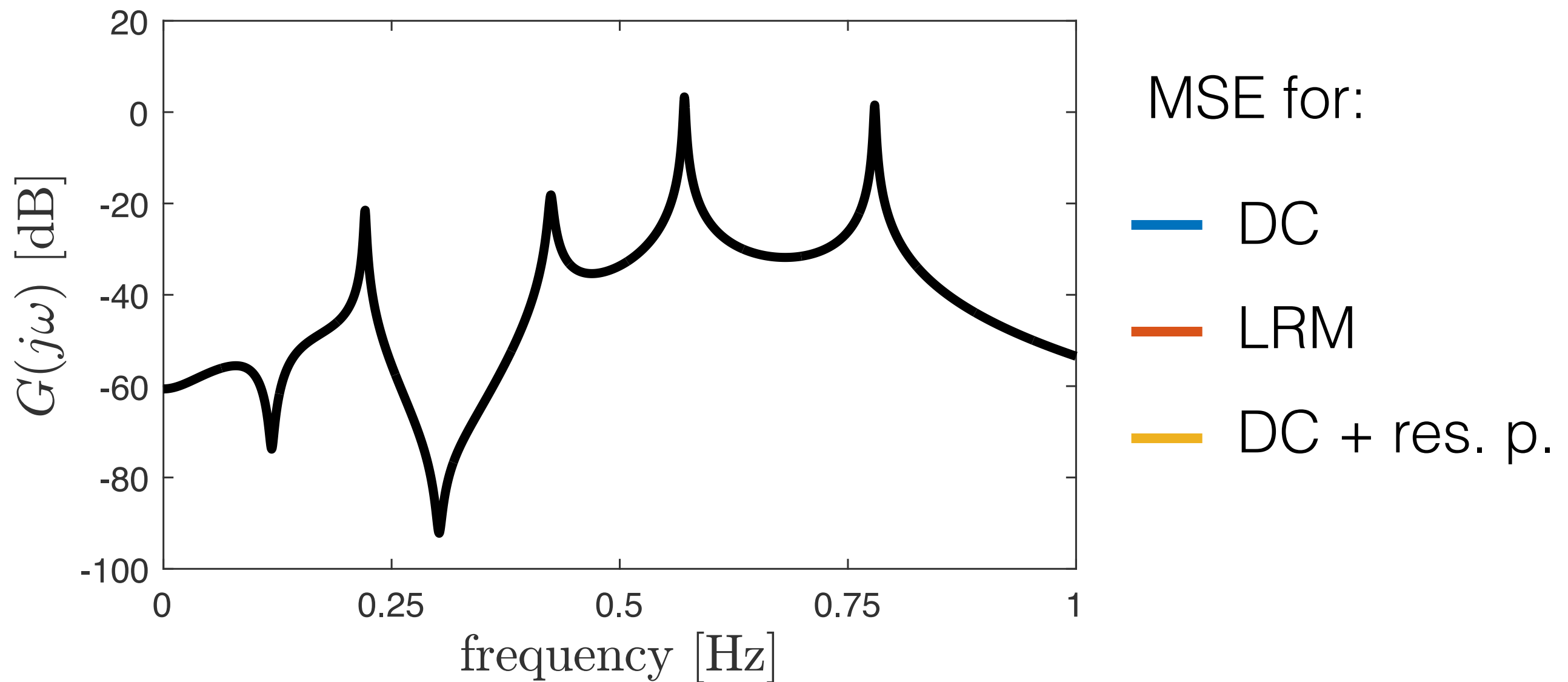
LRM $\rightarrow$ Estimate Poles



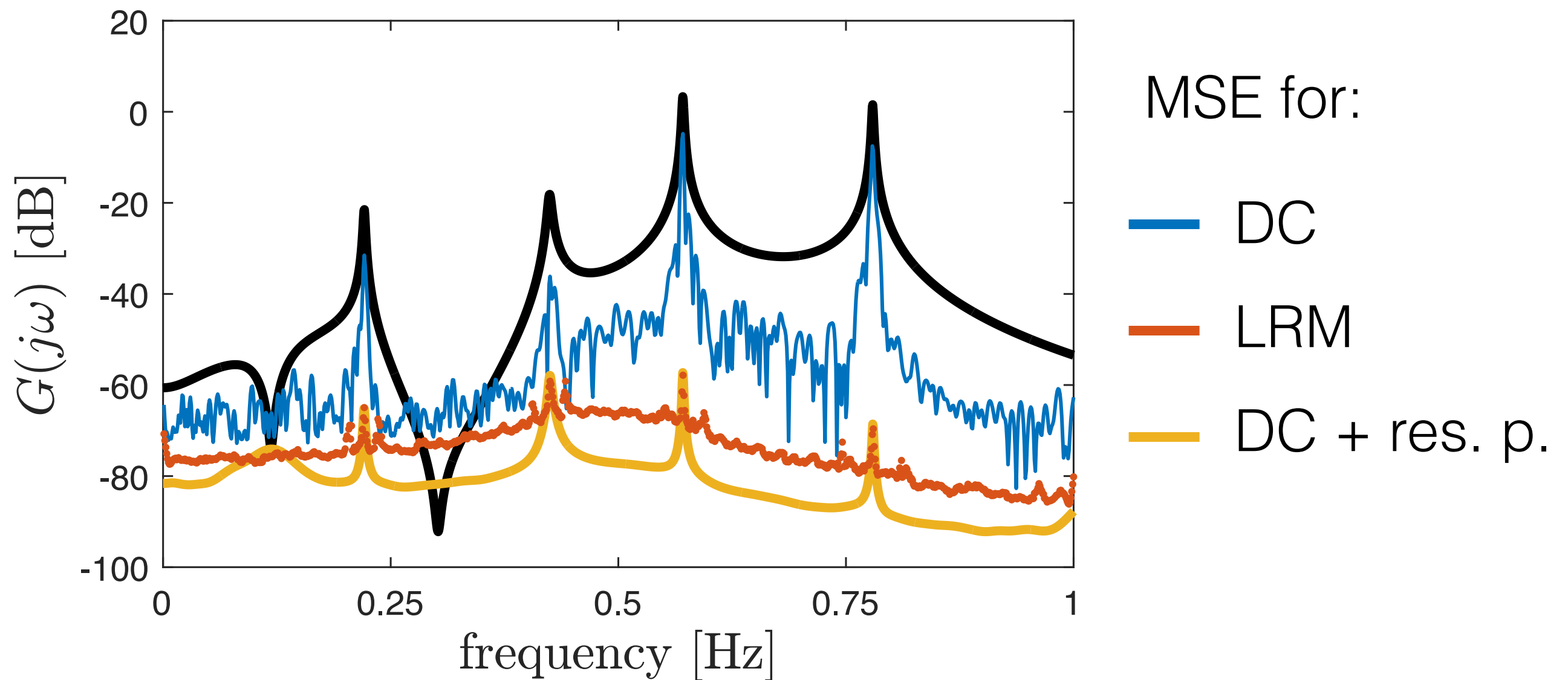
Kernel with estimated pole locations

$$k_{\text{total}}(t, \tau) = k_{\text{DC}}(t, \tau) + k_{\text{res},1}(t, \tau) + k_{\text{res},2}(t, \tau) + k_{\text{res},3}(t, \tau) + k_{\text{res},4}(t, \tau)$$

Results: DC + res. poles



Results: DC + res. poles



Kernel Methods

Transfer Functions

Frequency domain

Kernel Methods - Transfer Functions - Frequency domain

- Transients, for finite window lengths
- Frequency domain Kernel interpretations
- Complex GP, with relation matrix
- Prior knowledge on resonances from LRM

Main messages

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- System identification in the frequency domain
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 - select frequency band of interest
 - non-parametric models of coloured noise
- Nonlinear systems
- Time- and parameter varying systems
- Applications in a.o. HF/RF electronics